1992-09

Coupling losses in multiple beam antennas

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release; distribution is unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
Naval Postgraduate School

6b. OFFICE SYMBOL *(If applicable)*
EC

7a. NAME OF MONITORING ORGANIZATION
Naval Postgraduate School

6c. ADDRESS *(City, State, and ZIP Code)*
Monterey, CA 93943-5000

7b. ADDRESS *(City, State, and ZIP Code)*
Monterey, CA 93943-5000

8a. NAME OF FUNDING/SPONSORING ORGANIZATION

8b. OFFICE SYMBOL *(If applicable)*

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS *(City, State, and ZIP Code)*

10. SOURCE OF FUNDING NUMBERS

Program Element No.	Project No	Task No	Work Unit Accession Number

11. TITLE *(Include Security Classification)*
COUPLING LOSSES IN MULTIPLE BEAM ANTENNAS

12. PERSONAL AUTHOR(S) Randall J Tucker

13a. TYPE OF REPORT
Master's Thesis

13b. TIME COVERED
From To

14. DATE OF REPORT *(year, month, day)*
September 1992

15. PAGE COUNT
87

16. SUPPLEMENTARY NOTATION
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17. COSATI CODES

FIELD	GROUP	SUBGROUP

18. SUBJECT TERMS *(continue on reverse if necessary and identify by block number)*
Multiple Beam Antennas, Coupling Losses, Array Antennas, Multiple Beam Arrays

19. ABSTRACT *(continue on reverse if necessary and identify by block number)*

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT
[X] UNCLASSIFIED/UNLIMITED [ ] SAME AS REPORT [ ] DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
David C. Jenn

22b. TELEPHONE *(Include Area code)*
646-2254

22c. OFFICE SYMBOL
EC/JN

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted
All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

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Coupling Losses
in
Multiple Beam Antennas

by

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1992

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The relationship between beam coupling losses in multiple beam antennas and pattern parameters such as beamwidth and sidelobe level is investigated. A computer algorithm to calculate the coupling coefficients for arbitrary number of beams is implemented using MATLAB. Gain-loss data for various array amplitude distributions is presented.

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ACKNOWLEDGMENTS

I would like to thank my advisor, Professor Jenn for his patience and valuable assistance when my motivation was low and the learning curve was steep. I would also like to thank the Electrical and Computer Engineering faculty who took a keen interest in a physicist, and showed him that engineers really have all the fun. Finally, I take this opportunity to give a special thank you to my wife, Cindy, and my three children, Kim, Chris, and Samantha. Without their love, support, and understanding, this undertaking would not have been possible. This final product is as much theirs as it is mine.

Well done to all.

I. INTRODUCTION

An important component of all communications, radar, and electronic warfare systems utilizing the radio-frequency portion of the electromagnetic spectrum is the antenna. The antenna is a transition structure that serves to minimize the signal loss between a guiding structure and the transmission medium. It must efficiently radiate power when transmitting and efficiently collect power when receiving, and is usually called upon to give directional information as well. With so much reliance on the electromagnetic spectrum for detection, targeting, and communications, it is imperative that technical personnel understand the roles and design constraints of antennas.

There are three principal types of antenna systems in use:

1. Reflectors
2. Lenses
3. Arrays

Although multiple beams can be formed using any of these antennas, the analysis presented here will concentrate on arrays. An array is simply a collection of small antennas, generating a directional beam using constructive and destructive interference [Ref. 1:p. 204]. Array antennas are widely used in radar, and communications applications. Naval applications include the AN/SPY-1 radar for the AEGIS system, the AN/AWG-9 F-14 radar, and the receive multiple beam antennas for the AN/SLQ-32 electronic warfare system. Additionally, a conventional reflector monopulse radar consists of a 4 feed horn assembly arranged in a planar array.

Arrays are particularly well suited to the radar role for the following reasons:

1. High data rates are possible with electronic beam scanning.
2. Multiple beams can be generated.
3. Lower sidelobe levels are possible for Low Probability of Intercept Radar (LPIR).
4. Solid State technology can be efficiently utilized.

Before array technology, increasing directivity meant increasing the electrical size of the antenna relative to the wavelength of the transmitted signal. The problem of increasing the electrical size of an antenna was particularly exacerbated at lower frequencies. Arrays were necessitated since it was not feasible to generate a single element with the required dimensions. For example, at a frequency of 30 Megahertz (MHz), the corresponding wavelength is 10 meters ($\approx$ 30 ft). A two wavelength antenna would be 60 feet long. Alternatively, the same directivity could be obtained using 2 or 3 quarter wavelength antennas each 2.5 meters long as an array, with half-wavelength spacing. The efforts of pioneers like Schelkunoff led to new advances in antenna design using array techniques and polynomial methods.

In addition to electronically scanning the array with a single beam, it may be advantageous to receive multiple beams simultaneously. Multiple beam antennas offer the capability to scan sectors rapidly with little reduction in antenna gain. As mentioned, many systems currently in use by the United States Navy take advantage of multiple beam antenna technology, including the AN/SPY-1 radar, the AN/SPS-48 radar, and the AN/SLQ-32 Electronic Warfare suite.

The desire to optimize multiple beam systems has led to new synthesis techniques. Advances in computing ability and efficiency have led to various methods for generating multiple receive beams. These beam forming networks, with a matrix interpretation are well suited for implementation using array elements as feeds.[Ref. 2:p. 241] The advantages offered by phased array radar systems (fast scanning, low sidelobes, ECCM capability) combine with the advantages of multiple beam systems to further enhance overall system performance. Currently, most modern systems that employ multiple beam antennas rely on some type of array to implement the design.

A. MOTIVATION

High performance radar systems need narrow "agile" beams to accurately track targets. Many simultaneous beams are desired to track multiple targets. The necessity for high data rates common to most current weapons control systems makes electronic beam scanning necessary (as opposed to mechanical beam scanning). Given that premise, phased array antennas are the antennas of choice in most of these new systems.

Recent advances in Monolithic Microwave Integrated Circuits (MMIC) technology have expanded the number of RF components available in a small package. Currently, emphasis has been placed on integrating all of the RF devices necessary for a system (i.e.

phase shifters, radiating elements, low noise amplifiers, etc.) onto a single substrate. These technological advances have produced compact arrays with a large number of multiple beams.

B. PROBLEM STATEMENT

Intuitively, we would expect interaction between the various beams in a multiple beam system. For example, if one of the beam ports is excited, some of the energy will couple to the remaining beam ports and appear as a received signal. This coupled energy results in a loss in gain that would not be present if the antenna only had a single beam, and occurs even if there is perfect isolation between the beam feeds. This interaction has been investigated by many researchers, most notably Seymour Stein.[Ref. 3:p. 548] These beam coupling losses are an important consideration in the design and analysis of multiple beam antennas. There is a tradeoff between these losses and the beam crossover level. High crossover (i.e. closely spaced beams) are particularly desirable from a systems perspective. However, the beam coupling losses at these crossover levels may be excessive. In receive antennas, an adequate signal-to-noise ratio is maintained using low noise amplifiers (LNA's). The location of the amplifiers in the antenna feed and their required gain are influenced by the beamforming losses. Thus prediction of the beam coupling losses is essential as the number of multiple beams, and the crossover level increases. Figure 1 shows the radiation pattern for a typical multiple beam antenna array using a Taylor distribution

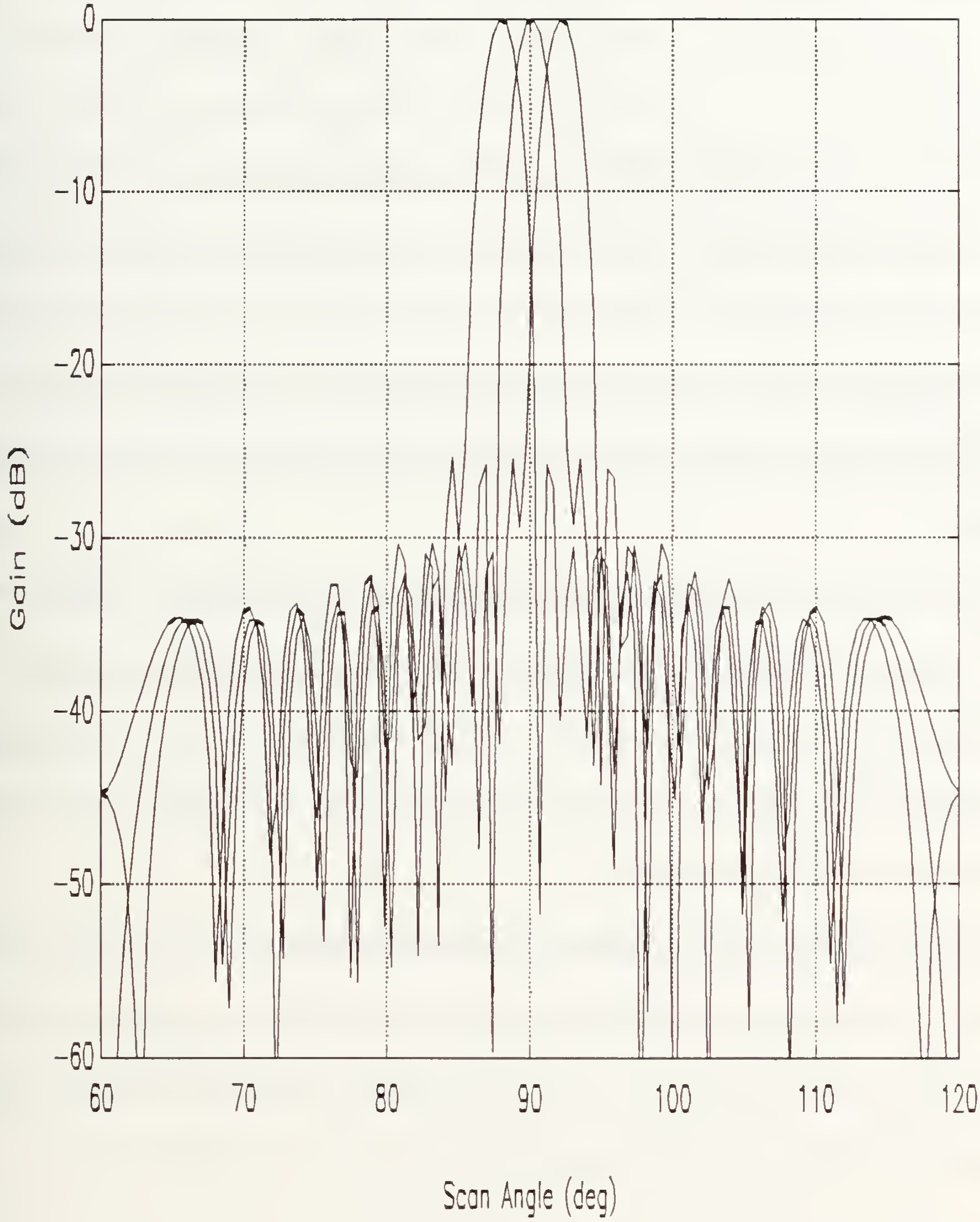

Figure 1. 3 Beam Multiple Beam Array - Taylor Distribution

with 25 dB sidelobes and $\bar{n}=5$. In this figure, there are 3 beams and the beams crossover at the 3 dB point.

C. APPROACH

Typically, beam coupling losses are estimated from curves such as those by Stein or from crude estimates obtained by considering only two beams and extrapolating to a higher number of beams. These techniques are effective for a few beams, but accuracy rapidly drops as the number of beams increases. Once an antenna development model is available, beam coupling losses can be measured, but this technique is costly and time consuming.

Stein has formulated the loss calculation in terms of a cross correlation integral and antenna scattering parameters. His formulation has reduced the problem to solving a matrix eigenvalue equation. Flexible software computing packages such as MATLAB™ or MATHEMATICA™ can be used to solve the equations for a wide range of geometry and excitation with a single algorithm.

The goal of this thesis is to compute the beam coupling losses in a multiple beam linear array as a function of the number of array elements, the element spacing, the beam crossover level, and the sidelobe level. For computational convenience it is assumed that the excitation coefficients for all beams are the same (the case of identical beams).

D. CHAPTER OVERVIEW

A breakdown of contents by chapter is provided here.

Chapter II lays the requisite mathematical foundation in antenna theory and gives a derivation of the fundamental relationships in array design.

Chapter III gives the mathematical foundation of the theory of beam coupling losses using a scattering matrix representation. The adaptation of Stein's work to linear arrays is also presented.

Chapter IV presents the summary of the calculated data for the two and four beam cases, and looks at the relationship between beam coupling losses for a given geometry and the number of beams.

Chapter V summarizes the work performed and gives the conclusions.

Analytical details, raw data, and computer source codes are included in the appendices.

II. ARRAY FUNDAMENTALS

One definition of an array is "... an assembly of radiating elements in an electrical and geometrical configuration."[Ref. 1:p. 204] Arrays produce radiation patterns using the principles of constructive and destructive interference. Destructive interference is used to suppress radiation in unwanted directions. The interference patterns are a function of the weighting of the individual elements. The weighting can in general be complex, and determines the beamwidth, sidelobe level, and scan direction [Ref. 4:p. 1]. The design parameters that influence the characteristics of the radiation pattern are:

1. geometrical configuration,
2. relative element separation,
3. element amplitude weight,
4. element phase weight,
5. radiating element pattern [Ref. 1:pp.204-205].

Maxwell's equations are used to derive the expression for the radiation pattern. Two assumptions that apply to antenna problems are

1. All quantities have a time-harmonic dependence. Phasor notation will be used with the $e^{j\omega t}$ suppressed.
2. Antenna surfaces are perfect electrical conductors (i.e., only $\bar{J}_s$).

The electric field at a field point P(x,y,z) due to the current $\bar{J}_s(x',y',z')$ can be expressed as [Ref. 1:pp.86-91]

$$\vec{E}(x,y,z) = -j\omega\vec{A} - \frac{j}{\omega\mu_0\varepsilon_0}\nabla(\nabla\cdot\vec{A}), \tag{1}$$

where

$$\vec{A}(x,y,z) = \frac{\mu_0}{4\pi}\iint_S \vec{J}_s(x',y',z')\frac{e^{-jkr}}{r}\,ds' \tag{2}$$
$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \quad .$$

S denotes all of the antenna surfaces; for an array it becomes a collection of identical discrete surfaces. Combining (1) and (2) and simplifying leads to the expression for the electric field

$$\vec{E}(x,y,z) = -\frac{j\omega\mu_0}{4\pi}\left\{\iint_S \vec{J}_s(x',y',z')\frac{e^{-jkr}}{r}ds' + \frac{1}{\omega\varepsilon_0}\nabla\left[\nabla\cdot\iint_S \vec{J}_s(x',y',z')\frac{e^{-jkr}}{r}ds'\right]\right\}. \tag{3}$$

In the far field $R\to\infty$, and the second term in (3) can be neglected since it will involve terms decreasing as $1/R^2$ and $1/R^3$. Using the geometry of Figure 2, and making the far-field approximation that $\bar{R}$ and $\bar{r}$ are parallel gives $|\bar{r}| \approx |\bar{R}| - \bar{R}' \cdot \hat{R}$. The electric field at P becomes

$$\vec{E}(x,y,z) = -\frac{j\omega\mu_0}{4\pi R}e^{-jkR}\iint_S\left[\vec{J}_s - (\vec{J}_s\cdot\hat{R})\hat{R}\right]e^{-jk\vec{R}'\cdot\hat{R}}\,ds, \tag{4}$$

where $\hat{R}$ is a unit vector in the direction of point P.

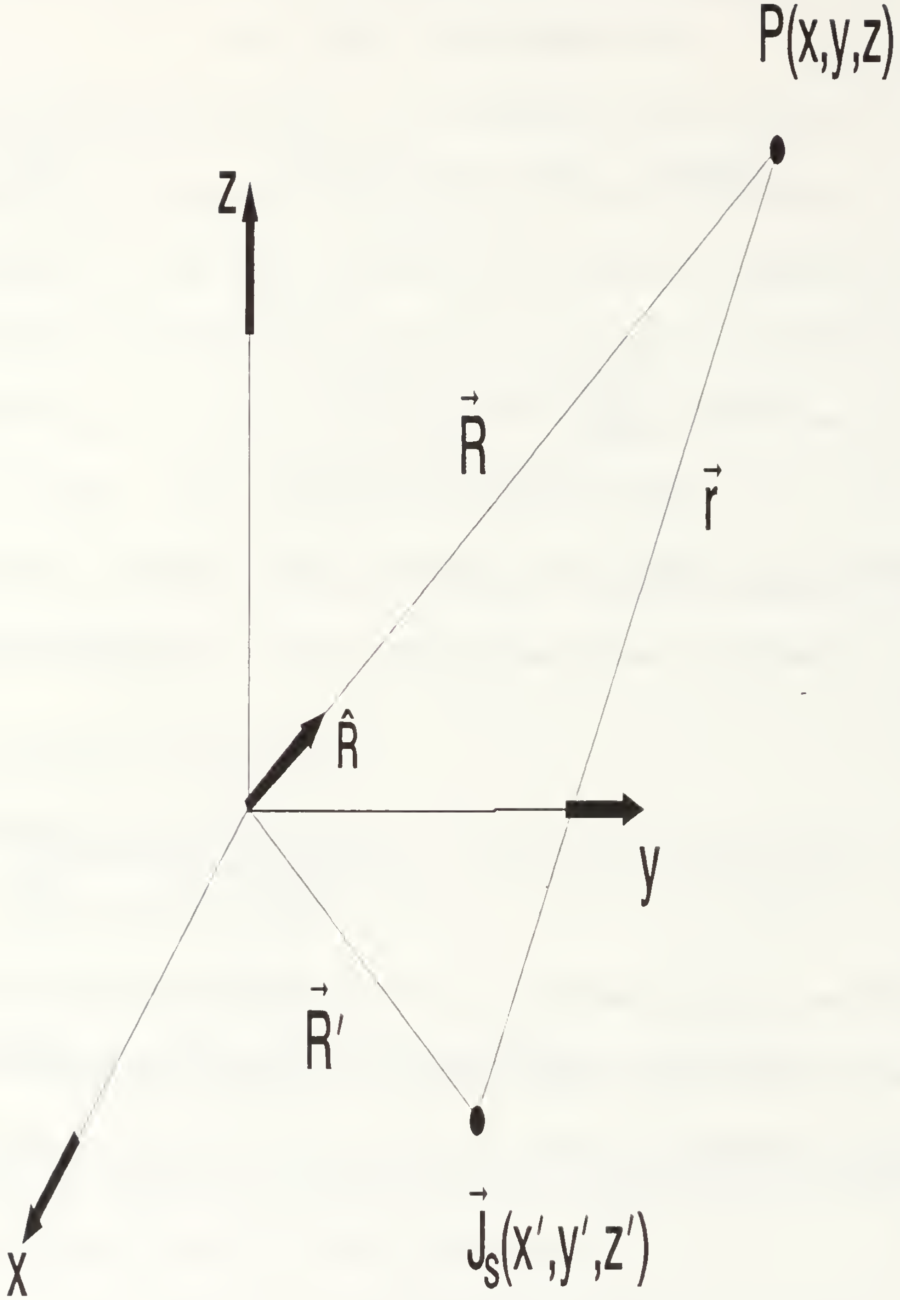

Figure 2. E Field Calculation Geometry (Far-Field)

Now that the expression for the electric field has been derived, it is necessary to extend this result to multiple elements. Consider an array consisting of N identical surfaces, and define a reference element with surface S_0 as shown in Figure 3. The expression for the electric field of the array can be written as

$$\vec{E}(x,y,z) = -\frac{j\omega\mu_0}{4\pi R}e^{-jkR}\sum_{i=1}^{N} c_i \iint_{S_0}\left[\vec{J}_{S_0} - \left(\vec{J}_{S_0}\cdot\hat{R}\right)\hat{R}\right]e^{jk\left[\vec{d}_i\cdot\hat{R}+\vec{R}_0'\cdot\hat{R}\right]}ds' \quad , \tag{5}$$

where c_i is a complex constant that relates the current on the i^{th} element ($\vec{J}_{s_i}$) to the current on the reference element ($\vec{J}_{s_0}$). By factoring the complex exponential term in the integral, (5) can be rewritten as

$$\vec{E}(x,y,z) = \left[\sum_{i=1}^{N} a_i e^{jk\vec{d}_i\cdot\hat{R}}\right]\left[\frac{-j\omega\mu_0}{4\pi R}e^{-jkR}\iint_{S_0}\left[\vec{J}_{S_0} - \left(\vec{J}_{S_0}\cdot\hat{R}\right)\hat{R}\right]e^{jk\vec{R}_0'\cdot\hat{R}}ds'\right] \tag{6}$$

$$= \quad (AF) \cdot (EF) \quad .$$

From (6), it can be seen that the total radiation pattern for the array is the product of the element radiation pattern (EF) and the array factor (AF, the term involving the summation). This is known as the principle of Pattern Multiplication.

For radar applications, the antenna gain, beamwidth, and sidelobe level are all important design parameters. High gain increases radar range and a narrow beam improves target resolution. However, low sidelobe levels have currently become a priority for clutter reduction and electronic counter counter-measures (ECCM). Unfortunately, low sidelobes are achieved at the expense of both gain and beamwidth. Thus, there is a relationship between beamwidth, sidelobe level, and gain that forms a

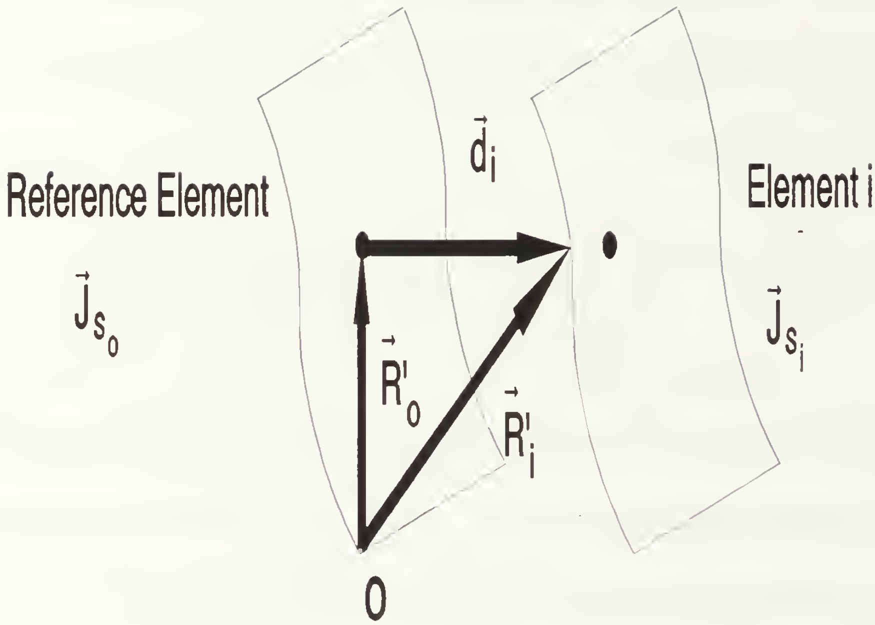

Figure 3. Array Element Geometry

classic engineering tradeoff. The sidelobe level is set by the proper choice of the magnitude of the constants, c_i. Table 1 gives a comparison between the beamwidth and sidelobe level of several commonly used amplitude windows.

TABLE 1. BEAMWIDTH - SIDELOBE COMPARISON

Window ($\lvert c_i \rvert = a_i$)	**Sidelobe Level** (dB)	**HPBW** (λ/L)
Uniform	-13.2	.886
Cosine	-23.0	1.188
Taylor (20 dB $\bar{n}=4$)	-20.0	1.188
Hamming	-42.0	1.302

The Taylor weighting is used extensively in radar applications, and is specified by two parameters: sidelobe level (SLL) and $\bar{n}$. For a given sidelobe level, it offers minimum beamwidth. Once either gain or SLL is specified, the other quantity is established.[Ref. 1:p.680]

Thus far only the effects of amplitude weighting the array elements have been discussed. However, as stated previously, the element weighting can be complex. The effects of phase weighting will now be addressed. Let the N isotropic radiators lie along the z-axis with element spacing d as shown in Figure 4. According to (6), the array factor is the linear superposition of the contributions of each individual element. In terms of the quantities in Figure 4, the array factor is

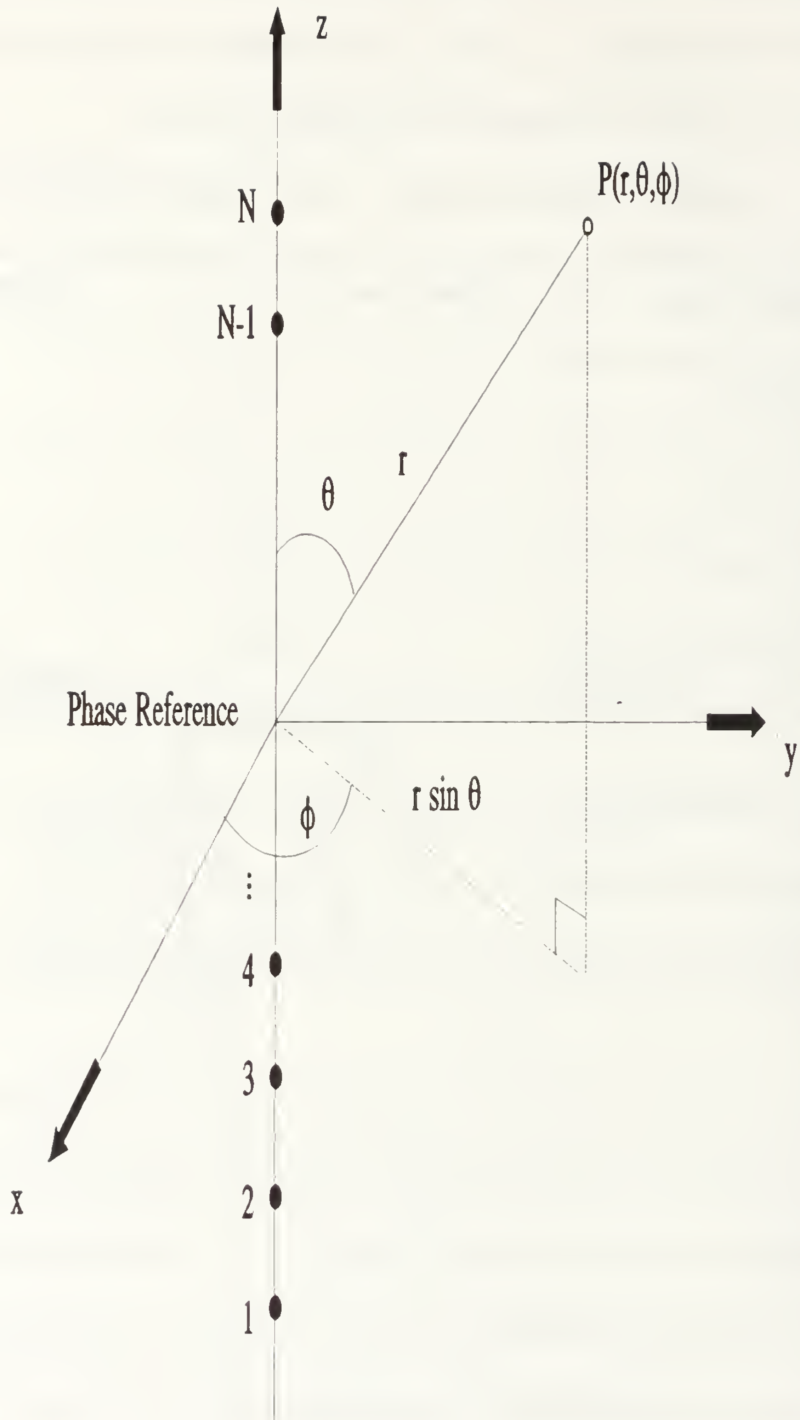

Figure 4. N-Element Array Far-Field Geometry

$$AF(\theta,\phi) = \sum_{n=1}^{N} a_n e^{j(kd\cos\theta + \beta)(n - \frac{N+1}{2})}. \tag{7}$$

Implicit in equation (7) is the representation of the complex weights as

$$c_n = a_n e^{+j\beta n}, \tag{8}$$

where a_n is the amplitude weight and β is a progressive phase shift between array elements. If $a_n=1$, the array is a uniform array, and the sum becomes a geometric series that can be reduced to the closed form

$$AF(\psi) = \frac{1}{N}\left[\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right], \tag{9}$$

where ψ=kd cos $\theta+\beta$. If $\beta=0$, the direction of maximum radiation is perpendicular to the axis of the array elements, and the beam is referred to as a broadside beam. The pattern for a 10 element uniform array with d= $\lambda/2$ is shown in Figure 5. If β=kd, the direction of maximum radiation is in the direction of the array axis, and the beam is referred to as an endfire beam [Ref. 1:p. 218].

The advent of ferrite materials and high speed switching circuits have made scanning radar arrays possible. By applying a progressive phase shift to equation 1, the far field radiation pattern is shifted by an amount determined by the linear phase variation across the array. The beam peak location θ_0 for the amount of phase shift β is determined from

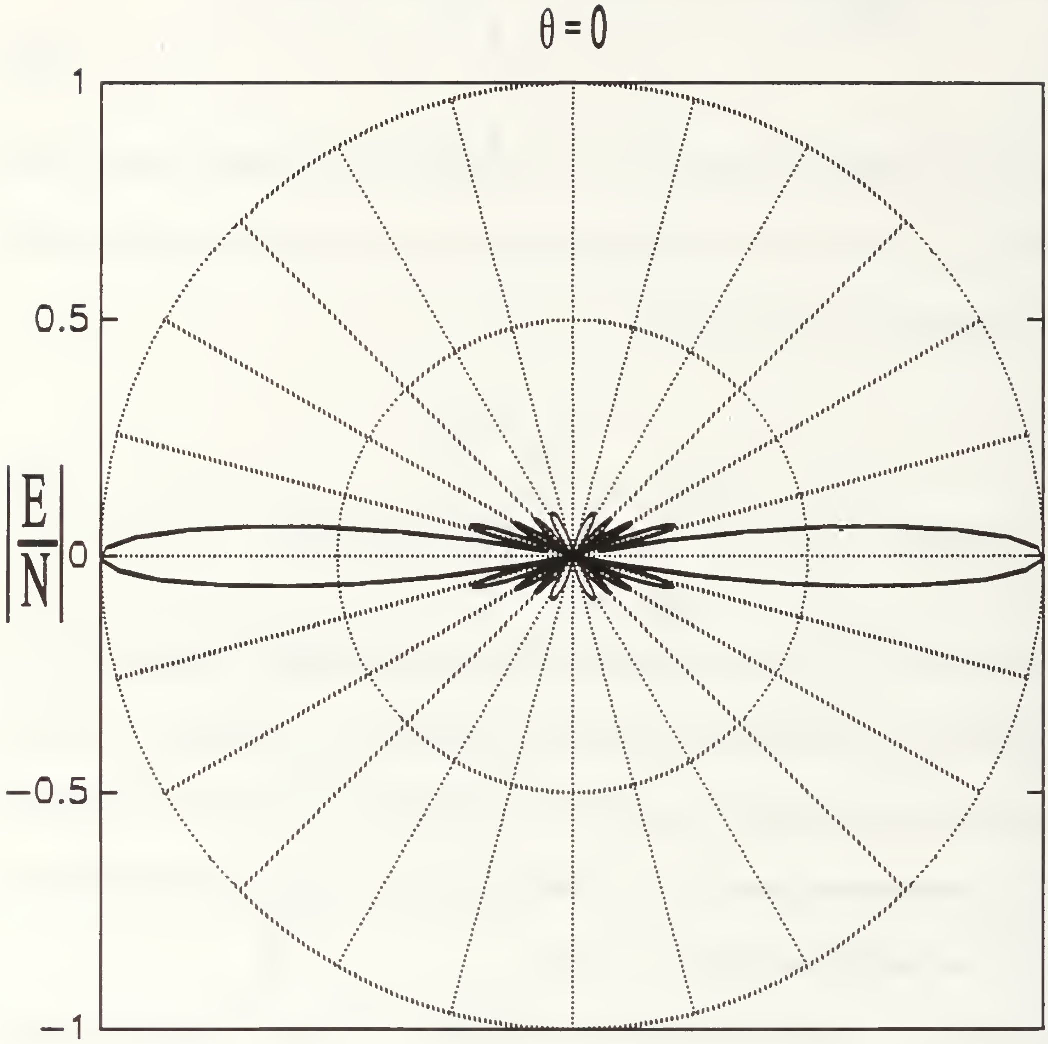

Figure 5. 10 Element Array Factor - Broadside Scan (15°/sector)

$$\beta = -kd\cos\theta_0 \quad . \tag{10}$$

Inserting this result into (7) and simplifying leads to

$$AF(\theta) = \sum_{n=1}^{N} |c_n| e^{j(n-1)kd(\cos\theta - \cos\theta_0)}, \tag{11}$$

where θ_0 is the desired scan angle. Broadside and endfire correspond to θ_0=90 and θ_0=0 or 180 degrees, respectively. For radiation in a particular direction between 0 and 90 degrees, the appropriate phase β must be selected according to equation (10). The array can then be made to scan in any desired direction, hence the term "phased array". [Ref. 1:p.220]

An example of a radiation pattern for a 10 element uniform array scanned to 45 degrees is shown in Figure 6. For most radar applications, the scan is predominantly ± 60° from broadside. Furthermore, only radiation into one hemisphere is desired. To achieve this, the linear array is placed above a ground plane. If the ground plane spacing is properly chosen, the net effect is to leave the pattern above the ground plane essentially unchanged and eliminate the radiation in the back hemisphere. Linear arrays above a ground plane can be modelled as a planar array radiating in free space by the method of images.[Ref.1:p. 137] The pattern for a 10 element uniform array over a ground plane scanned to 45 degrees is shown in Figure 7. The gain will increase by a factor of approximately 2 if the power into the antenna is kept the same, since all of the power is confined to a single hemisphere. Current systems that employ phased array

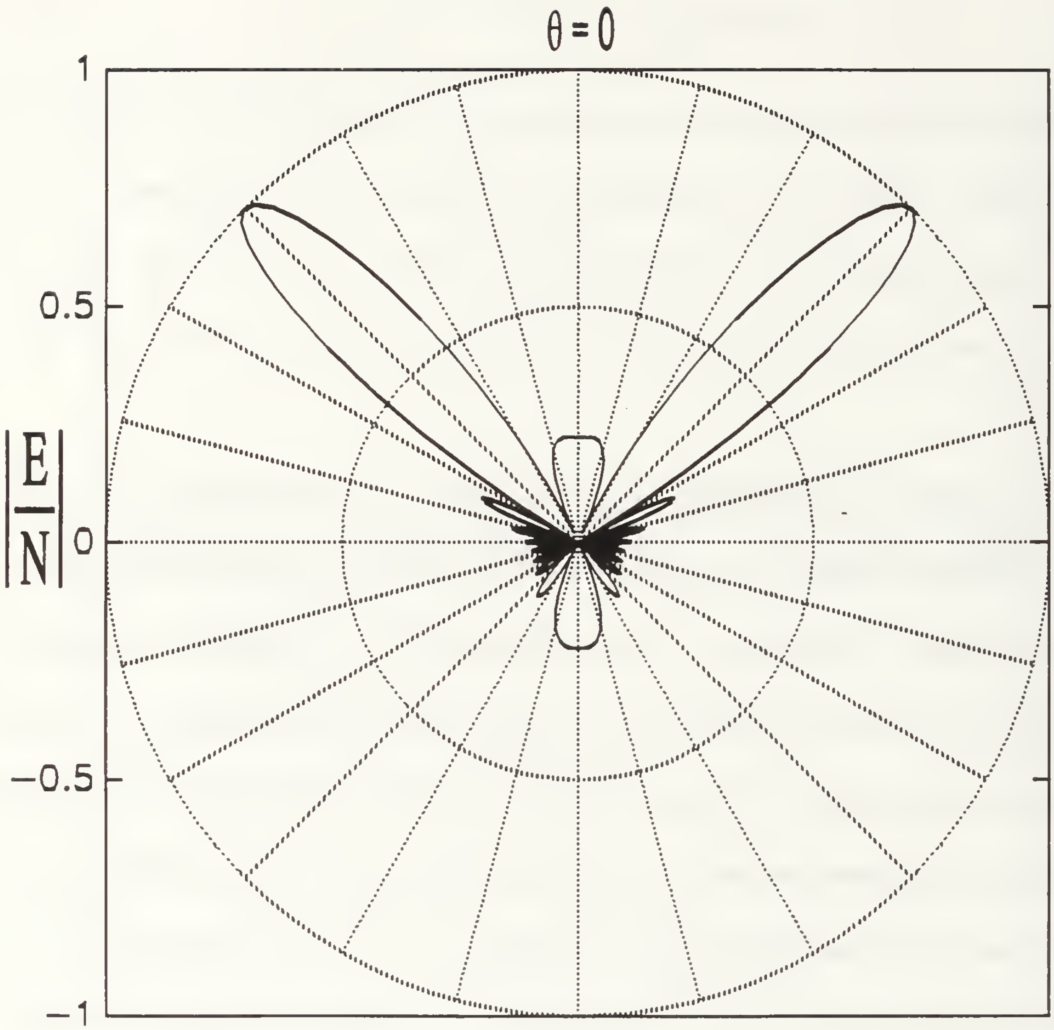

Figure 6. 10 Element Array Factor - 45 Degree Scan
(No Ground Plane)

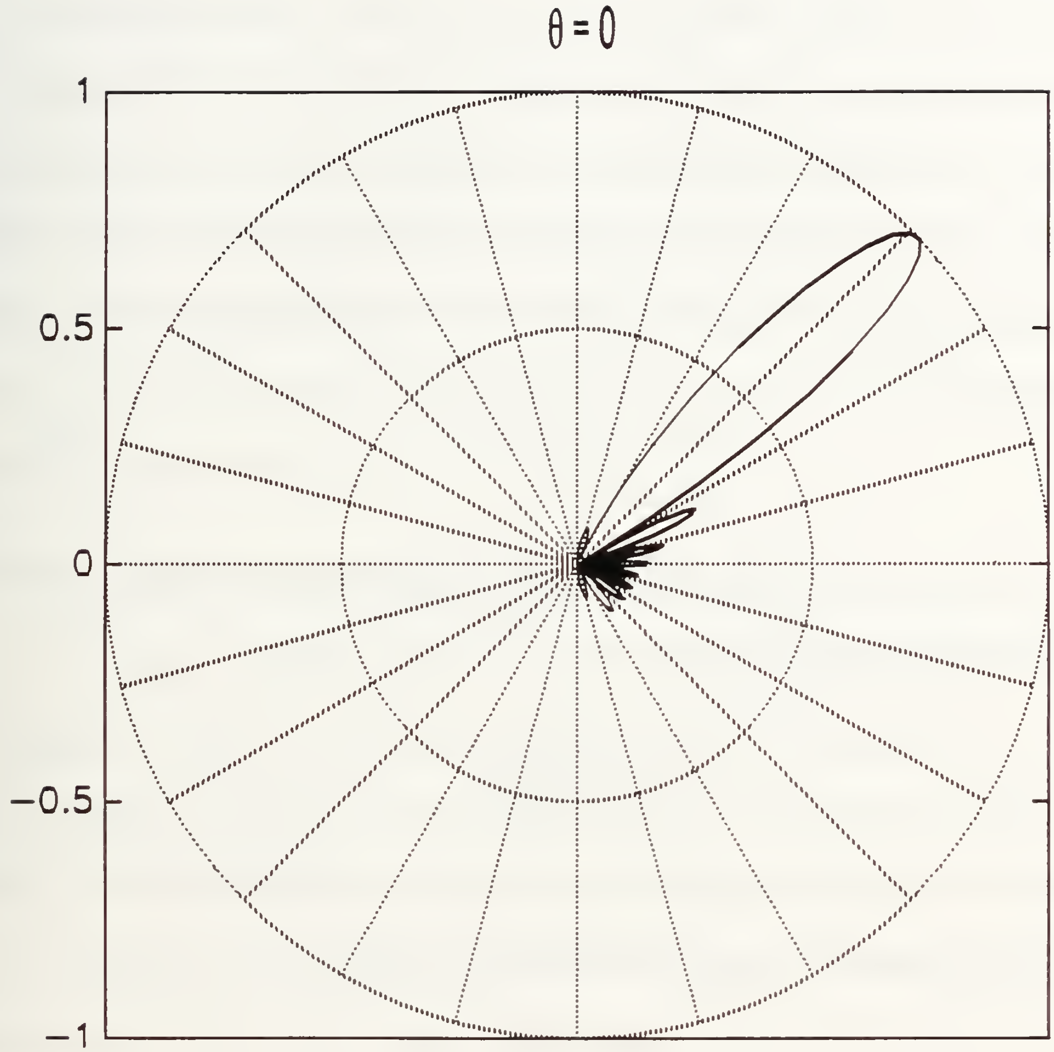

Figure 7. 10 Element Array Over Ground Plane - 45° Scan (15°/Sector)

technology include the AEGIS AN/SPY-1 series of Naval Radars, the PATRIOT Air Defense Weapons System, and the PAVE PAWS Early Warning Radar System.

The final array design parameter of interest is the interelement spacing, d. When scanning an array, multiple directions of maximum radiation within the visible region ($0 \leq \theta \leq 180$ degrees) may arise. These additional maxima are not desirable and are referred to as grating lobes [Ref. 5:p. 114]. The choice of d influences the number and location of these grating lobes. The grating lobes appear when ψ is an even multiple of 2π. Accordingly, the locations of the grating lobes are given by

$$\cos\theta_g = \cos\theta_0 + \frac{m\lambda}{d} \qquad m = \pm 1, \pm 2, \cdots \quad . \tag{12}$$

To ensure that no grating lobes appear in the visible region requires that $\cos\theta_g > 1$, and thus the limitation on spacing becomes

$$d < \frac{\lambda}{1 + |\cos\theta_o|} \quad . \tag{13}$$

When $\theta_0 = 0$, $\cos\theta_0 = 1$ and when $\theta_0 = \pi$, $|\cos\theta_0| = 1$. Therefore if $d < \lambda/2$, no grating lobes will appear for any scan angle θ_0 [Ref. 6:p.144].

III. MATHEMATICAL FORMULATION OF BEAM COUPLING

A. INTRODUCTION

An antenna can be modelled as a multiport junction with scattering parameters S_{ij}. The S_{ij} are computed from a knowledge of the beam radiation patterns. Once the beam radiation patterns and the S_{ij} are known, the radiation efficiency can be determined by solving a matrix eigenvalue equation. This solution technique will be specialized to the case of a phased array antenna system. The model is based on the work of Stein [Ref. 2], and his notation will be used.

B. SCATTERING MATRIX FOR N-PORT NETWORKS

For electrical networks at low frequencies ($f < 20$ khz), the standard lumped parameter circuit models work well in describing the behavior of the circuit. The circuit can be described by voltages across and currents through each element. The voltages and currents are related by the element impedances and admittances. However, relationships for low frequencies are not valid for microwave frequencies, since the phase of the electromagnetic wave may change significantly over the length of the element.[Ref. 7:p.1] The lack of the standard circuit models at microwave frequencies makes measurement of voltage and current extremely difficult. For this reason, at microwave frequencies it makes more sense to work with the incident and reflected fields in a network. The use of incident and reflected (or scattered) fields leads to an equivalent network formulation, called the scattering matrix representation.[Ref. 7:p.221] Scattering matrices mathematically describe the behavior of waves at each port of a device. For an N-port network,

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} , \tag{14}$$

where y denotes an outgoing wave and x an incoming wave. The elements S_{ij} are called the scattering parameters of the device. In matrix form

$$y = S x \quad . \tag{15}$$

The S_{ij} are found by driving port j with an incident wave, and measuring the outgoing wave at port i, with all other ports terminated in matched loads.

It is possible to visualize a multiple beam antenna as an N-port network consisting of N input feeds and N output beams. If the network is reciprocal (as is usually the case for most antennas that transmit and receive), the scattering matrix will be symmetric. [Ref. 7:p.224] The scattering matrix can be related to other matrix formulations, such as the impedance or admittance matrix. As with the latter two, the scattering matrix provides a complete representation of antenna performance[Ref. 7:p.223]. Figure 8 gives a conceptual picture of a multiple beam antenna system represented as an N-Port network.

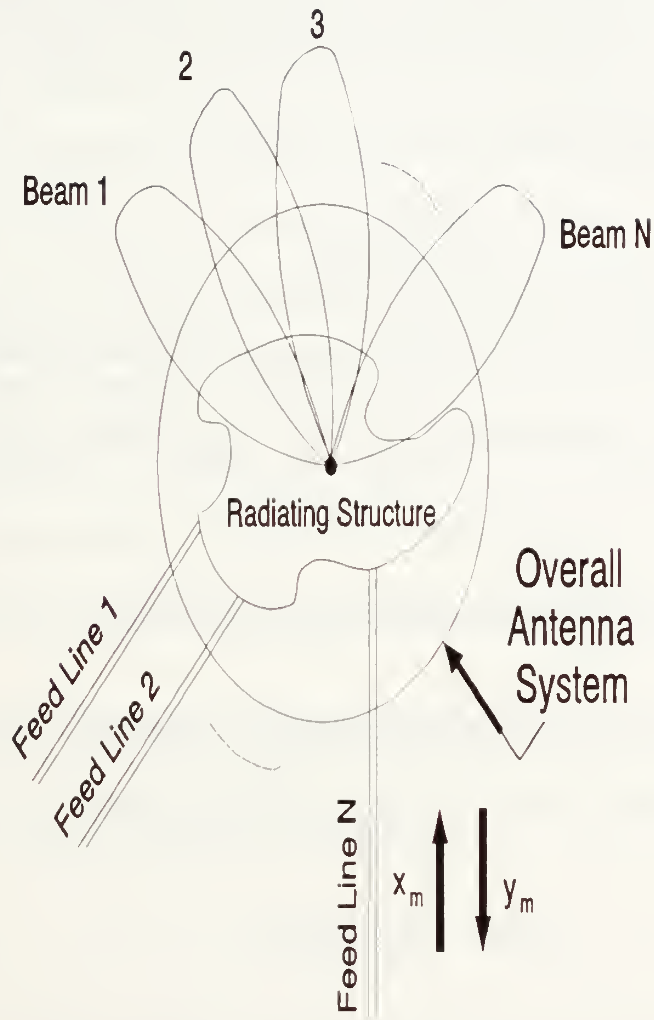

Figure 8. N-Port Network (after Stein)

C. BEAM COUPLING FACTORS

Using the notation in Stein's paper [Ref. 2], and treating the antenna assembly as an N-port network, the incident fields at each port are represented by the vector **x**, and the reflected fields at each port are represented by the vector **y**. The x_k and the y_k are related via the scattering matrix

$$y_k = \sum_{m=1}^{N} S_{km} x_m \quad . \tag{16}$$

The incident waves give rise to far-field beam patterns and the far-field radiation patterns are normalized using an autocorrelation integral over the spatial angular domain as follows

$$\int_0^{2\pi} \int_0^{\pi} \vec{R}_k^*(\theta,\phi) \cdot \vec{R}_k(\theta,\phi) \sin\theta \, d\theta \, d\phi = 2\eta_0 \quad , \tag{17}$$

where $\bar{\mathbf{R}}_k(\theta,\phi)$ is the beam pattern of the k[th] beam ,η_0 is the impedance of free space (120π Ω), and the * denotes complex conjugate. The electric fields are of the form

$$\vec{E}_k(\theta,\phi) = q_k \vec{R}_k(\theta,\phi) \frac{e^{jkr}}{r} \quad . \tag{18}$$

The term q_k is equivalent to the square root of the radiation efficiency and r is the distance from the local antenna coordinate system origin to the observation point.

The beam coupling coefficients are given by

$$\beta_{kj} = \frac{1}{2\eta_0}\int_0^{2\pi}\int_0^{\pi} \vec{R}_k^*(\theta,\phi)\cdot\vec{R}_j(\theta,\phi)\sin\theta\, d\theta\, d\phi \quad . \tag{19}$$

The coupling coefficients possess symmetry in k and j ($\beta_{kj}=\beta_{kj}^*$) and are a measure of the beam overlap. Since the $\bar{\mathbf{R}}_k(\theta,\phi)$ are vectors with complex components (i.e, each having a magnitude and phase), it is not just the area common to the polar diagrams of the multiple beams. The complete overlap of beams k and j implies $\beta_{kj}=1$; if $\beta_{kj}=0$, the beams are orthogonal. For uniform illumination, orthogonality occurs when beams cross at approximately the 4 dB points.

Having established a measure of coupling, it is natural to investigate the implications of conservation of energy. If the antenna system is treated as a linear network, superposition can be used to compute the total radiated electric field.

$$\vec{E}(\theta,\phi) = \left[\sum_{k=1}^{N} x_k q_k \vec{R}_k(\theta,\phi)\right] \frac{e^{jkr}}{r} \quad . \tag{20}$$

Using equation (16), the incident total power P_{inc} is

$$\begin{aligned} P_{inc} &= \sum_k x_k^* x \\ &= \boldsymbol{x}^\dagger \boldsymbol{x} \end{aligned} \quad , \tag{21}$$

where the † indicates a conjugate transpose. Similarly the reflected power is

$$P_{ref} = \boldsymbol{x}^\dagger \boldsymbol{S}^\dagger \boldsymbol{S}\boldsymbol{x} \quad . \tag{22}$$

The total radiated power, P_{rad}, is defined as

$$P_{rad} = \frac{1}{2}\int_0^{\pi}\int_0^{2\pi} r^2 \eta_0 \vec{E}^*(\theta,\phi)\cdot\vec{E}(\theta,\phi)\sin\theta\, d\theta\, d\phi \quad . \tag{23}$$

Appropriate substitution and simplification leads to an alternative expression for P_{rad},

$$\begin{aligned} P_{rad} &= \sum_{k=1}^{N}\sum_{j=1}^{N} x_k^* q_k^* \beta_{kj} q_j x_j \\ &= \sum_{k=1}^{N}\sum_{j=1}^{N} x_k^* \Gamma_{kj} x_j \quad . \end{aligned} \tag{24}$$

Conservation of energy now requires that

$$P_{inc} \geq P_{ref} + P_{rad} \quad , \tag{25}$$

where equality holds only if the network is purely lossless.

D. SPECIALIZATION TO AN ARRAY

1. COUPLING COEFFICIENTS

The adaptation of Stein's theory to linear arrays as presented here follows original work documented in reference [8] [Ref. 8:p. 1]. The first step is to determine the normalization constant for the k^{th} beam in the array. The array element spacing is d, and the amplitude distribution is given by the coefficients $\{a_n\}$. The magnitude of the normalization constant for the p^{th} beam is defined as

$$|A_p| = \sqrt{\frac{2kd}{\eta_0 \pi}} \left| \sum_{m=1}^{N}\sum_{n=1}^{N} a_m a_n I_{nm} e^{jkd\cos\theta_p (m-n)} \right|^{-\frac{1}{2}} \quad . \tag{26}$$

If $m=n$, $I_{nm}=2kd$; otherwise,

$$I_{nm} = 2kd\left\{\frac{\sin[(n-m)kd]}{(n-m)kd}\right\} \quad . \tag{27}$$

The beam coupling coefficients are computed using (19). Using the notation in Chapter 2, the array factor is substituted for the beam pattern, $\bar{\mathbf{R}}_k$. Neglecting the element factor will not have a significant effect if the radiating elements are well matched over the range of beam scan angles. Thus,

$$\beta_{pq} = \frac{\eta_0}{2}\int_0^{\pi}\int_{-kd}^{kd}\frac{A_p^* A_q}{kd}\left[\sum_{m=1}^{N} a_m e^{-jQ(m)(u-u_p)}\right]\left[\sum_{n=1}^{N} a_n e^{jQ(n)(u-u_q)}\right]du\,d\phi \tag{28}$$

with $Q(m)=\{2m-(N+1)\}/2$. The a_m and a_n are the amplitude coefficients for the beams, the u_p and u_q are k times the phase shifts to scan the p^{th} and q^{th} beams, and u is the z direction cosine times k. Performing the integration and simplifying leads to the final expression for the coupling coefficient

$$\beta_{pq} = \frac{\pi e^{j\frac{N+1}{2}(u_p-u_q)}\displaystyle\sum_{m=1}^{N}\sum_{n=1}^{N} a_m a_n e^{j(u_p m - u_q n)} I_{nm}}{\left|\displaystyle\sum_{m=1}^{N}\sum_{n=1}^{N} a_m a_n e^{ju_p(m-n)} I_{nm}\right|^{\frac{1}{2}}\left|\displaystyle\sum_{m=1}^{N}\sum_{n=1}^{N} a_m a_n e^{ju_q(m-n)} I_{nm}\right|^{\frac{1}{2}}} \quad . \tag{29}$$

2. RADIATION EFFICIENCY AND COUPLING LOSS

The calculation of the beam coupling loss is based on conservation of energy. Stein has shown that the problem can be cast into the form of a matrix eigenvalue equation. From the results of section (C(1)), the conservation of energy equation can be written as

$$x^{\dagger}x \geq x^{\dagger}S^{\dagger}Sx + x^{\dagger}\Gamma x \quad . \tag{30}$$

Both of the matrices, $\mathbf{\Gamma}$ and $\mathbf{S}^{\dagger}\mathbf{S}$, are Hermitian and positive semidefinite. Thus, from linear algebra, each has N positive real or zero eigenvalues. Each matrix also has a full set of linearly independent eigenvectors. It is possible to construct a matrix, **U**, from the normalized eigenvectors described above that will diagonalize $\mathbf{\Gamma}$ and $\mathbf{S}^{\dagger}\mathbf{S}$. If **U** is used to diagonalize $\mathbf{\Gamma}$, the transformation is a similarity transformation and has the mathematical form of

$$U^{\dagger}\Gamma U = \gamma \quad , \tag{31}$$

where γ is a diagonal matrix. **U** is also a unitary matrix, since it has the mathematical property

$$U^{\dagger}U = I \quad , \tag{32}$$

where **I** is the identity matrix.

Returning to the energy conservation equation, (30) can be transformed using the matrix **U**. Let $\mathbf{x}=\mathbf{Uz}$; substitution of this result into equation (30) gives

$$z^{\dagger}z \geq z^{\dagger}U^{\dagger}S^{\dagger}SUz + z^{\dagger}U^{\dagger}\Gamma Uz \quad . \tag{33}$$

The transformation from x to z is equivalent to exciting the antenna with a new set of voltages. Since **x** was arbitrary to start with, the new vector **z** represents an equally arbitrary set of voltages. Rewriting (33) gives

$$z^{\dagger}(I - \gamma)z \geq z^{\dagger}(U^{\dagger}S^{\dagger}SU)z \quad , \tag{34}$$

in which $U^{\dagger}S^{\dagger}SU$ is also Hermitian and positive semidefinite.

The characteristic equation of (34) is

$$|I - \gamma| = 0 \quad . \tag{35}$$

Most standard mathematical software packages contain routines that can efficiently solve this eigensystem. The result is N eigenvalues $(\lambda_1, \lambda_2, \cdots, \lambda_N)$ that are related to the beam efficiencies ($|q_1|^2$, $|q_2|^2$, $\cdots$, $|q_N|^2$) by

$$\gamma_k = |q_k|^2 \beta_{kk} = \frac{1}{\lambda_k} \quad . \tag{36}$$

Because $\beta_{kk} = 1$ by choice of normalization,

$$|q_k|^2 = \frac{1}{\lambda_k} \quad . \tag{37}$$

Since $|q_k|^2 \leq 1$ from conservation of energy arguments, the upper bound on the radiation efficiency is given by

$$|q|^2_{max} = \frac{1}{\lambda_{max}} \quad . \tag{38}$$

For the special case of identical beams, $|q_1| = |q_2| = \cdots = |q_N|$, the determinant for the radiation efficiency factor becomes

$$\begin{vmatrix} (1-\lambda) & \beta_{12} & \cdots & \beta_{1N} \\ \beta_{21} & (1-\lambda) & \cdots & \beta_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{N1} & \beta_{N2} & \cdots & (1-\lambda) \end{vmatrix} = 0 \quad . \tag{39}$$

Using the symmetry of the coupling coefficients, allows the equation to be written as

$$\begin{vmatrix} (1-\lambda) & \beta_{21}^{*} & \cdots & \beta_{N1}^{*} \\ \beta_{21} & (1-\lambda) & \cdots & \beta_{N2}^{*} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{N1} & \beta_{N2} & \cdots & (1-\lambda) \end{vmatrix} = 0 \quad . \tag{40}$$

The evaluation of the determinant analytically proceeds by cofactor expansion. Expanding and collecting terms leads to the final result of

$$\lambda^{N} + A_{1}\lambda^{N-1} + A_{2}\lambda^{N-2} + \cdots + A_{N-1}\lambda + A_{N} = 0 \quad . \tag{41}$$

The largest root of this equation, λ_{max}, is then substituted into (38) to obtain $|q_{max}|^2$. The detailed solution for four beams is given in Appendix B. The results from Appendix B were used to verify the computer code which is based on the more general eigenvalue solution of equations (35) through (38). Note that identical beams can only be achieved where there is perfect symmetry, such as a circular array or a spherical array. However, the beams away from the edges of a large cluster of beams can be considered "approximately" identical.

IV. DATA SUMMARY

A. INTRODUCTION

The beam coupling loss data was computed for various amplitude weights applied to the array to control beamwidth (hence crossover level) and sidelobes. The amplitude weights used were

1. Uniform
2. Cosine on a 10 dB pedestal
3. Cosine on a 15 dB pedestal
4. Taylor (20 dB, $\bar{n} = 4$)
5. Taylor (30 dB, $\bar{n} = 5$)
6. Taylor (40 dB, $\bar{n} = 6$)

These distributions were chosen because of their practical use in radar and communication systems, and because of their ease of computer implementation. MATLAB has a built- in routine to calculate uniform weights, the Cosine and Taylor distributions were generated using author-created functions. Figure 9 shows samples of the various amplitude windows.

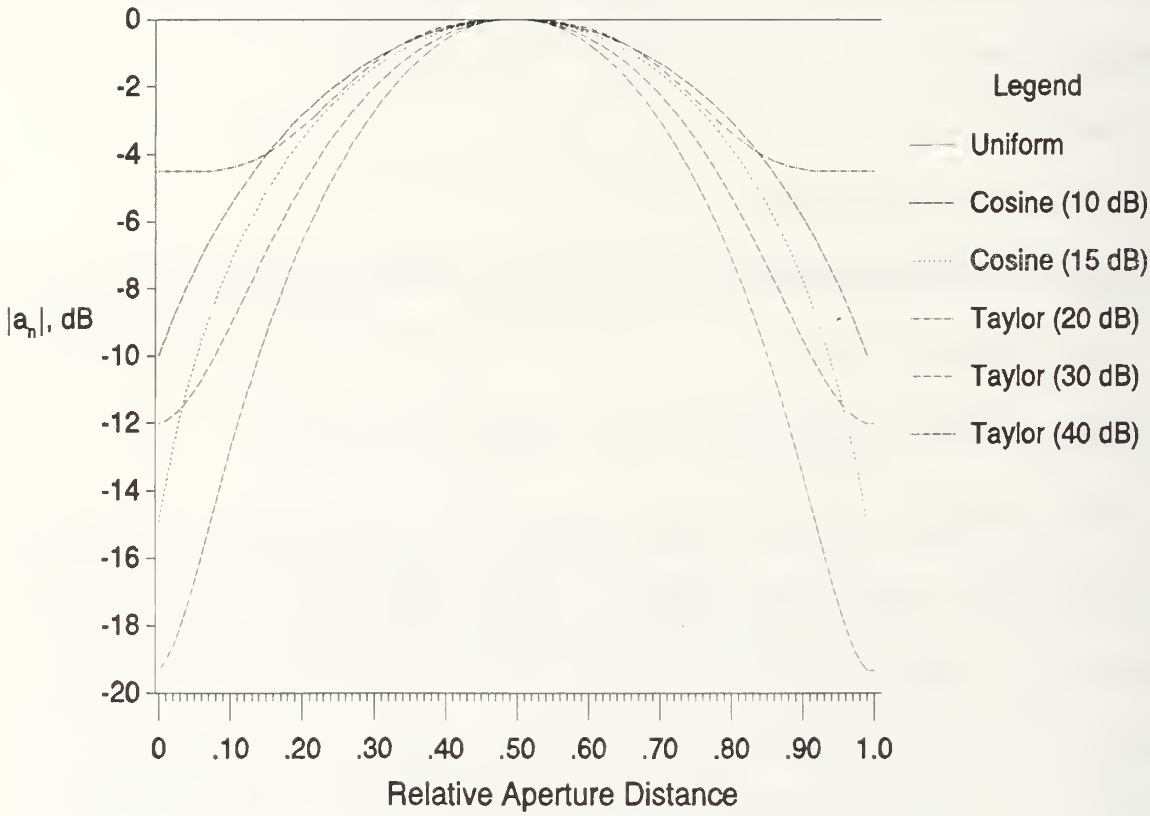

Figure 9. Amplitude Weightings

B. 2 BEAM DATA

This section presents the summary of data for the 2-beam array scenarios. The complete raw data is presented in a set of tables located in Appendix A.

The data summary consists of parametric plots of the coupling loss. With two simultaneous beams, the coupling loss is related to the crossover level through the coupling coefficient (β) between the beams

$$q^2 = \frac{1}{1 + |\beta|} \quad . \tag{42}$$

A plot of coupling loss vs crossover level for the 20 element array is shown in Figure 10. The plot contains the data for all six amplitude weightings. Figure 11 shows a similar plot for a 100 element array.

C. 4 BEAM DATA

Following is a summary of data for the 4 beam array scenarios. The complete data set is presented in tables located in Appendix A. As in the 2 beam case, the data summary consists of plots of the coupling loss. With four simultaneous beams, the coupling loss is related to the crossover level by

$$q^2 = \frac{1}{\lambda_{max}} \quad , \tag{43}$$

where λ_{max} is the largest eigenvalue of the β matrix. Plots of radiation efficiency vs crossover level for the 4 beam, 20 element array are shown in Figure 12. The plots

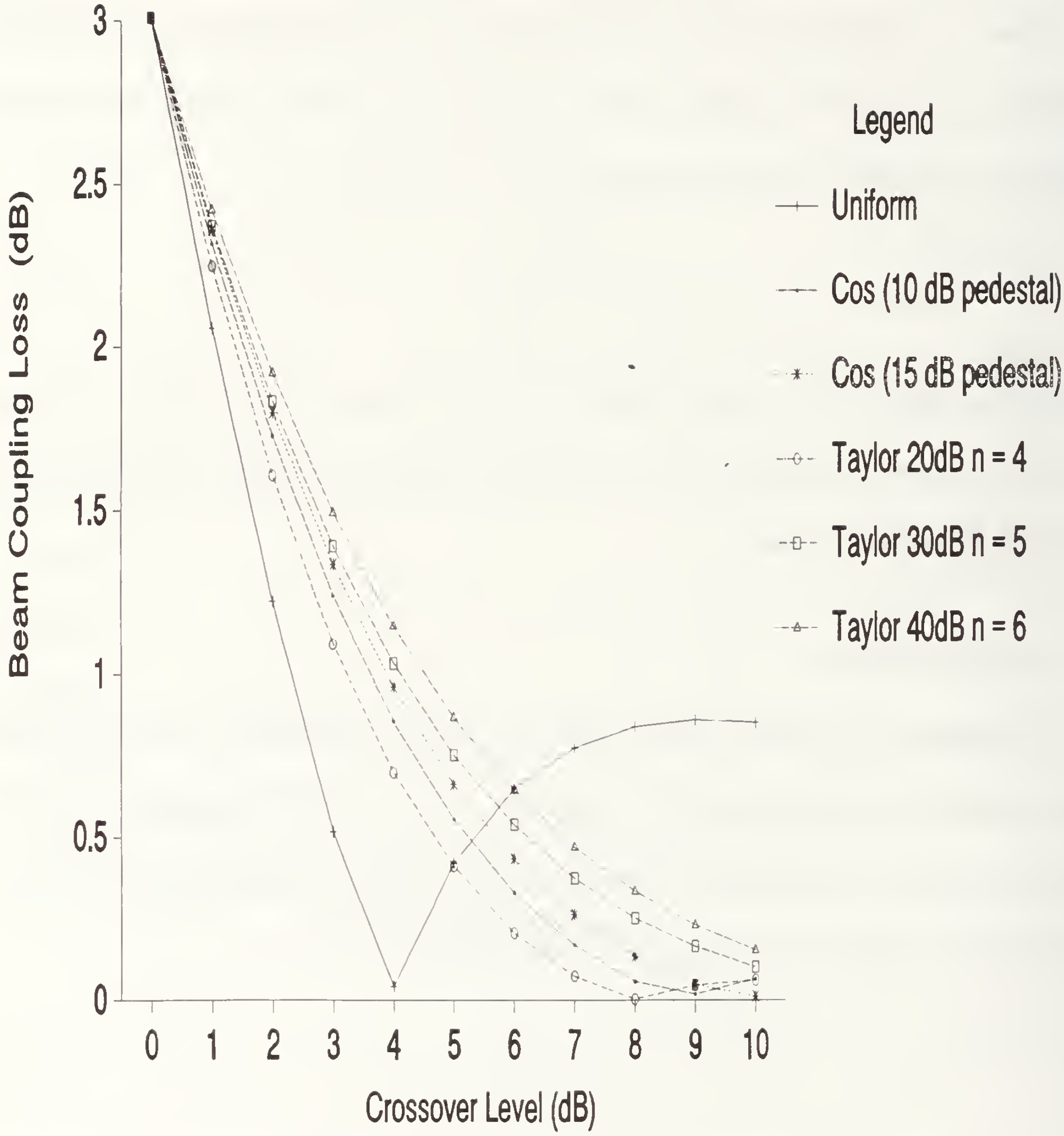

Figure 10. 2 Beam Coupling Loss - 20 Element Array

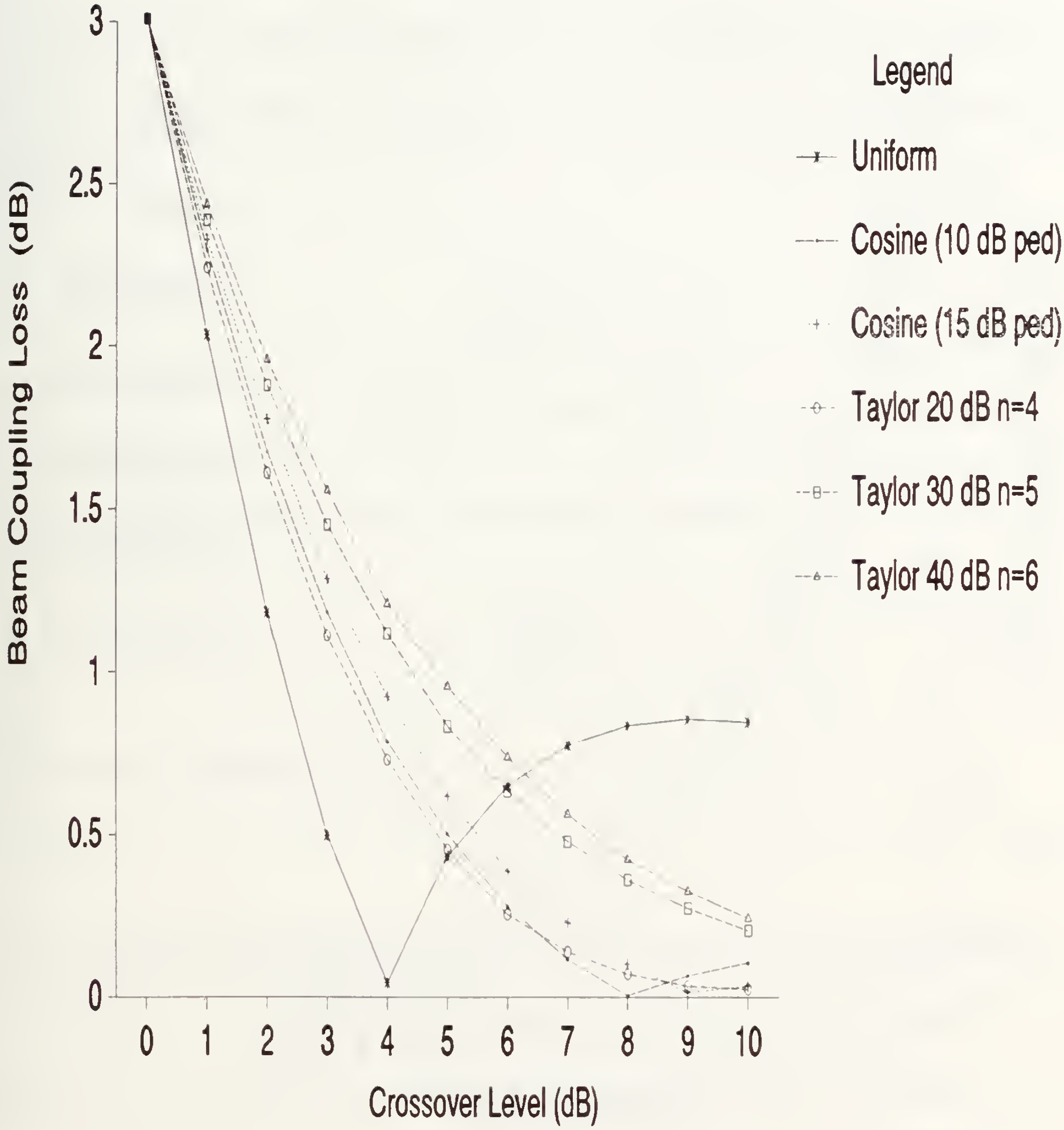

Figure 11. 2 Beam Coupling Loss - 100 Element Array

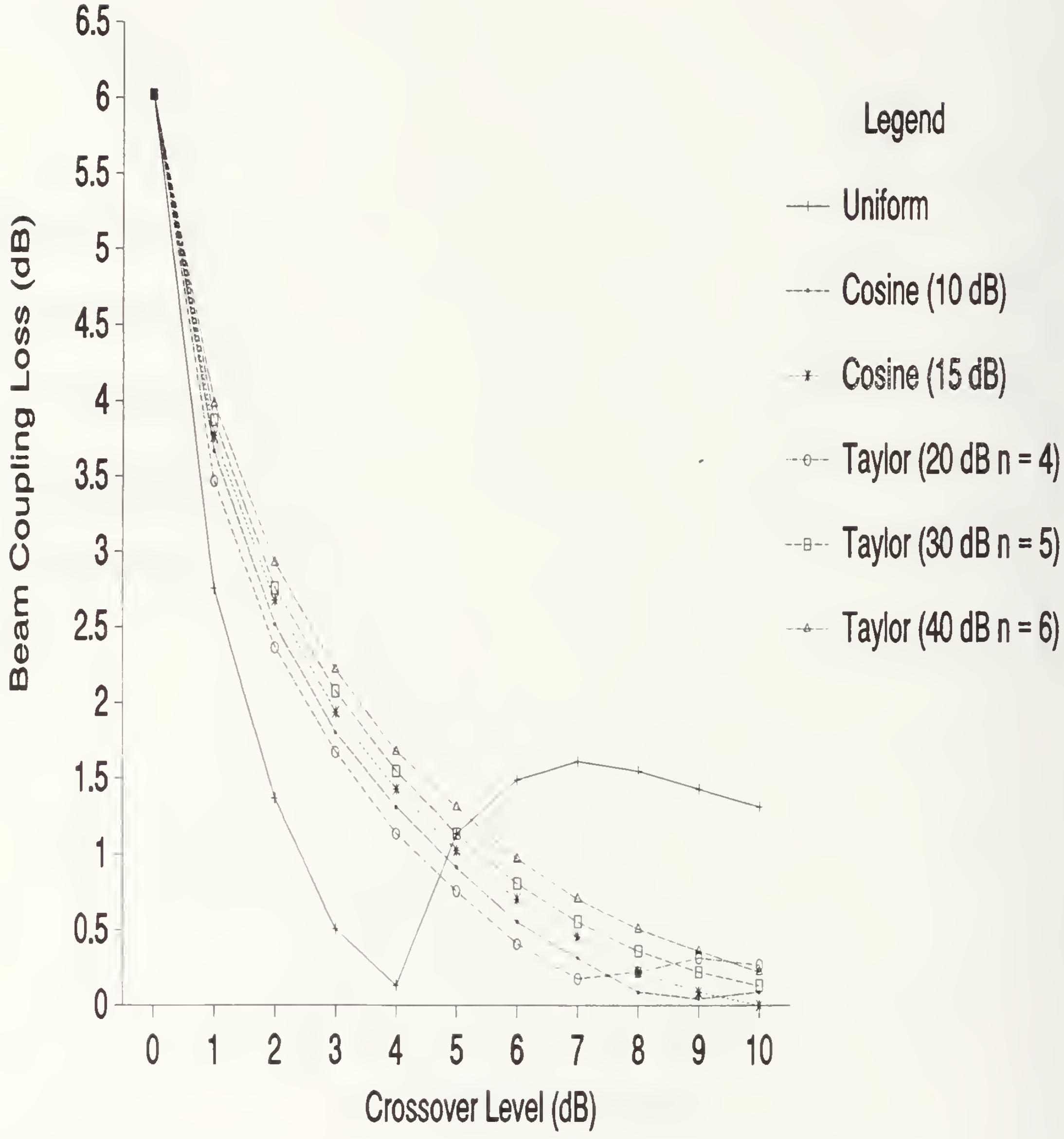

Figure 12. 4 Beam Coupling Loss - 20 Element Array

contain the data for all six amplitude weightings, and the corresponding plot for a 100 element array is shown in Figure 13.

D. MULTIPLE BEAMS AND GAIN LOSS

One important result of multiple beam antenna research is the investigation of gain loss due to beam coupling. According to Hansen, the gain for a linear array is [Ref. 9:p. 20].

$$G = \frac{\left(\sum_{n=1}^{N} a_n\right)^2}{\sum_{n=1}^{N}\sum_{m=1}^{N} a_n a_m \operatorname{sinc}(n-m)kd} \quad . \tag{44}$$

If the interelement spacing, d, is $\lambda/2$, equation (44) reduces to

$$G = \frac{\left(\sum a_n\right)^2}{\sum a_n^2} \quad , \tag{45}$$

which can also be expressed as

$$G = N\eta_a \quad , \tag{46}$$

where η_a is the aperture efficiency (i.e., gain loss due to non-uniform amplitude weights). This is the gain of an isolated lossless antenna with excitation coefficients $\{a_n\}$. (In standard antenna terminology, equation (46) gives the directivity.) In practice, other losses will further reduce the antenna efficiency. The losses include transmission line losses, mismatches in the feed, and the coupling losses if multiple beams are formed.

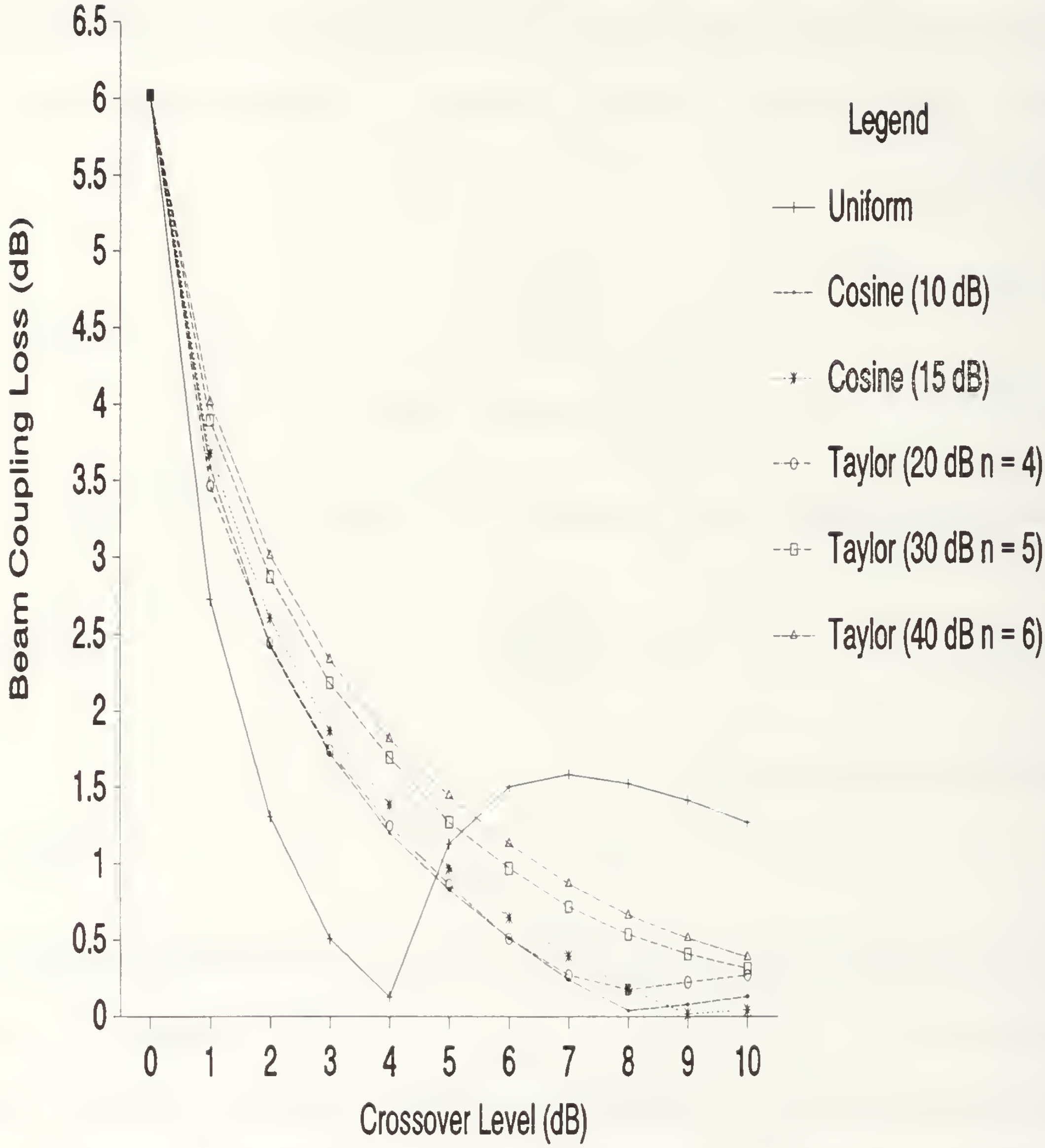

Figure 13. 4 Beam Coupling Loss - 100 Element Array

The overall antenna gain is determined by the product of the individual efficiencies. When only tapering and beam coupling losses are considered for beam k

$$G_k = N \eta_a |q_k|^2 \quad . \tag{47}$$

As an example, the aperture efficiency of a single beam array with a 30 dB Taylor distribution is .8733. When a second beam is added with a 3 dB crossover level, there is an additional .7258 reduction in gain. For N=20,

$$\begin{aligned} G &= (20)\,(.8733)\,(.7258) = 12.67 \\ &= 11\ dB \end{aligned} \tag{48}$$

Thus the beam coupling loss is 1.39 dB. It is clear that any design incorporating multiple beam techniques must account for the coupling losses to assure proper antenna operation from a systems perspective.

E. BEAM COUPLING LOSSES VS. NUMBER OF BEAMS

In the preceding sections, we have examined the effect of various antenna design parameters such as sidelobe level and beamwidth on beam coupling losses. At this juncture it is important to also investigate the effect of fixing design parameters and determining the relationship between beam coupling losses and the number of beams. For this investigation, only three scenarios will be investigated

1. Uniform Weighting
2. Cosine on a 10 dB Pedestal Weighting
3. Taylor 25 dB, $\bar{n} = 6$ Weighting

For each one of the amplitude weights listed, the beam coupling losses for each crossover level were compared for the center beams. Figure 14 shows a graph of the coupling loss versus the number of beams for a 25 dB Taylor distribution with a crossover level of 3 dB. The difference in coupling loss between the 2 beam case and the 6 beam case is less than 0.9 dB. The results indicate that the addition of a beam has little or no effect if its scan direction is greater than several beamwidths from the center beam. When this condition is true, the new entries in the β matrix due to the added beam are small, and the eigenvalues are primarily determined by the elements near the diagonal.

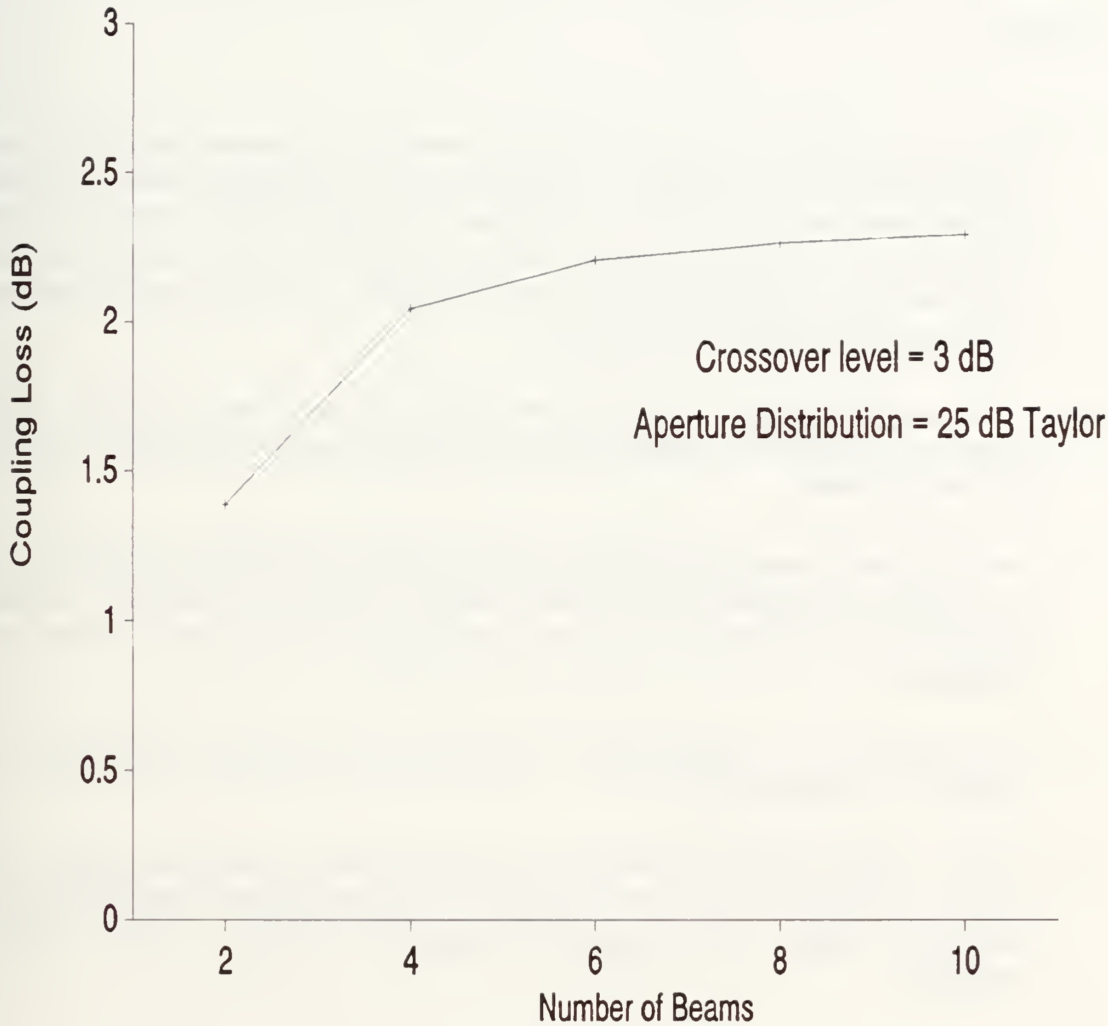

Figure 14. Coupling Loss vs Number of Beams

V. CONCLUSIONS

Analysis of the data presented in this report supports several conclusions. These conclusions are

1. For arrays, the pertinent loss equations are readily implemented and solved using commercial software packages. Availability of these packages makes it possible for personnel involved with array antennas (especially junior officers) to more completely understand why the equipment behaves as it does. A more detailed understanding of equipment performance translates to increased mission readiness and improved program management.

2. Beam coupling losses are dependent upon the basic array antenna design parameters and therefore must be traded off with beamwidth, sidelobe level, and gain requirements.

3. The reduction in array gain observed when using multiple beam techniques varies as a function of amplitude taper and crossover level, and is related to radiation efficiency. The reduction is not overwhelming, in most cases, neither is it insignificant.

4. The curves of the coupling loss vs crossover were distinctly different for low sidelobe and uniform arrays.

5. Future research should examine planar arrays and include the effect of the radiating element factor. The option of integrating the radiation patterns numerically (rather than summing the aperture coefficients) should also be considered.

Array antennas are in use throughout the radio frequency portion of the spectrum, and recent printed circuit arrays operate at frequencies as high as 800 Gigahertz (GHz) [Ref. 10]. Phased array technology will continue to grow in use due to its advantages in sidelobe suppression, scanning speed, and compactness. It is necessary to understand the benefits and limitations of this technology if it is to be exploited to its full potential.

APPENDIX A - CALCULATED DATA

A. 2 BEAM DATA

TABLE 2. 2 BEAM UNIFORM - 20 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
[illegible]	6.6000	0.0099	0.5000	3.0103
1	4.7200	0.6064	0.6225	2.0585
[illegible]	6.6000	0.3251	0.7547	1.2225
[illegible]	8.0000	0.1263	0.8879	0.5165
[illegible]	9.1300	0.0099	0.9902	0.0428
5	10.1000	0.1022	0.8879	0.4226
6	10.9400	0.1612	0.8612	0.6491
7	11.6500	0.1950	0.8368	0.7737
8	12.3200	0.2131	0.8243	0.8390
9	12.9100	0.2190	0.8203	0.8600
10	13.4200	0.2168	0.8218	0.8522

TABLE 3. 2 BEAM UNIFORM - 100 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.5000	3.0103
[illegible]	0.9540	0.5979	0.6258	2.0356
2	1.3321	0.3136	0.7613	1.1846
3	1.6022	0.1212	0.8919	0.4967
[illegible]	1.8183	0.0099	0.9902	0.4322
[illegible]	2.0164	0.1046	0.9053	0.4322
8	2.1785	0.1613	0.8611	0.6493
7	2.3226	0.1950	0.8368	0.7738
8	2.4487	0.2118	0.8252	0.8345
9	2.5568	0.2172	0.8216	0.8536
10	2.6650	0.2147	0.8232	0.8448

TABLE 4. 2 BEAM COSINE (10 dB PEDESTAL) - 20 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.5000	3.0103
4	5.5700	0.7050	0.5865	2.3172
8	7.8200	0.3209	0.6716	1.7287
4	9.5200	0.3307	0.7515	1.2408
4	10.8900	0.2182	0.8209	0.8572
5	12.0800	0.1367	0.8797	0.5565
6	13.1300	0.0792	0.9266	0.3310
7	14.0500	0.0399	0.9616	0.1699
8	14.8900	0.0133	0.8209	0.0574
9	15.6600	0.0043	0.9957	0.0186
10	16.3500	0.0152	0.9850	0.0655

TABLE 5. 2 BEAM COSINE (10 dB PEDESTAL) - 100 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.8909	3.0103
4	1.0981	0.6967	0.5894	2.2960
2	1.5482	0.4709	0.6799	1.6757
3	1.8723	0.3134	0.7614	1.1840
4	2.1425	0.1987	0.8342	0.7872
5	2.3587	0.1225	0.8909	0.5017
6	2.5568	0.0663	0.8342	0.2789
7	2.7370	0.0271	0.9736	0.1160
8	2.8992	0.0011	0.8989	0.0046
9	3.0434	0.0151	0.9851	0.0652
10	3.1696	0.0244	0.9761	0.1049

TABLE 6. 2 BEAM COSINE (15 dB PEDESTAL) - 20 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.5000	3.0103
4	5.9300	0.7216	0.9409	2.3593
2	8.3500	0.5136	0.6607	1.8001
4	10.1500	0.3603	0.7351	1.3363
4	11.6400	0.2477	0.8015	0.9611
5	12.9300	0.1648	0.8585	0.6625
8	10.0500	0.1052	0.9048	0.4344
7	15.0600	0.0628	0.9409	0.2645
8	15.9900	0.0320	0.9888	0.1368
9	16.8200	0.0113	0.9888	0.0488
10	17.5900	0.0030	0.9970	0.0130

TABLE 7. 2 BEAM COSINE (15 dB PEDESTAL) - 100 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.5000	3.0103
1	1.0981	0.7092	0.5851	2.3279
2	1.5482	0.5050	0.6645	1.7754
3	1.8723	0.3446	0.7437	1.2858
4	2.1425	0.2374	0.8082	0.9249
5	2.3587	0.1525	0.8677	0.6165
6	2.5568	0.0934	0.9146	0.3877
7	2.7370	0.0541	0.9487	0.2287
8	2.8992	0.0237	0.9769	0.1015
9	3.0434	0.0034	0.9966	0.0148
10	3.1696	0.0080	0.9920	0.0348

TABLE 8. 2 BEAM TAYLOR 20 dB $\bar{n}$ = 4 - 20 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.9093	3.0103
1	5.1900	0.6787	0.5957	2.2497
2	7.2900	0.0488	0.6906	1.6080
3	8.8600	0.2864	0.7774	1.0938
4	10.1300	0.1747	0.8513	0.6993
5	11.2100	0.0997	0.9093	0.4127
6	12.1900	0.0488	0.9535	0.2069
7	13.0400	0.0175	0.9828	0.0753
2	13.7900	0.0010	0.9990	0.0043
9	14.5000	0.0109	0.9892	0.0471
10	15.1300	0.0146	0.9856	0.0629

TABLE 9. 2 BEAM TAYLOR 20 dB $\bar{n}$ = 4 - 100 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.5000	3.0103
1	1.0441	0.6758	0.5967	2.2423
2	1.4582	0.4493	0.6900	1.6117
3	1.7643	0.2923	0.7738	1.1136
4	2.0164	0.1835	0.8450	0.7316
5	2.2326	0.1101	0.6908	0.4536
[illegible]	2.4307	0.0605	0.9429	0.2552
4	2.5929	0.0325	0.9685	0.1390
8	2.7370	0.0165	[illegible]	0.0711
9	2.8812	0.0080	0.9921	0.0344
10	3.0074	0.0057	0.9943	0.0249

TABLE 10. 2 BEAM TAYLOR 30 dB $\bar{n}$ = 5 - 20 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.5000	3.0103
1	5.8200	0.7268	0.5791	2.3724
2	8.1800	0.5254	0.6556	1.8338
3	9.9500	0.3777	0.7258	1.3915
4	11.4200	0.2689	0.[illegible]391	1.0343
[illegible]	12.6900	0.1891	0.8410	0.7522
6	14.8[illegible]00	0.1319	0.8835	0.5381
[illegible]	14.8000	0.0902	0.9173	0.3751
8	15.7100	0.0602	0.9432	0.2539
9	16.5400	0.0391	0.9624	0.1666
10	17.3100	0.0239	0.9767	0.1026

TABLE 11. 2 BEAM TAYLOR 30 dB $\bar{n}$ = 5 - 100 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
7	0	1.0000	0.5000	3.0103
1	1.0441	0.7329	0.5771	2.3879
2	1.4582	0.5415	0.6487	1.8794
3	1.7643	0.3969	0.7158	1.4518
3	2.0164	0.2937	0.7300	1.1182
5	2.2326	0.2114	0.8255	0.8330
6	2.4307	0.1571	0.8642	0.8330
7	2.5929	0.1164	0.8957	0.4782
8	2.7370	0.0866	0.9203	0.3607
8	2.8812	0.0651	0.9388	0.2740
10	3.0074	0.0482	0.9540	0.2044

TABLE 12. 2 BEAM TAYLOR 40 dB $\overline{n}$ = 6 - 20 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
5	0	1.0000	0.9480	3.0103
8	6.3500	0.7463	0.5726	2.4212
2	8.9300	0.5557	0.8189	1.9193
3	10.9000	0.4103	0.7091	1.4931
3	12.5200	0.3024	0.7678	1.1474
5	13.9200	0.2212	0.8189	0.8679
9	15.1700	0.1604	0.8618	0.6461
7	16.3100	0.4103	0.8974	0.4700
8	17.3500	0.0802	0.9258	0.3350
9	18.2900	0.0549	0.9480	0.2321
10	19.1700	0.0362	0.9651	0.1544

TABLE 13. 2 BEAM TAYLOR 40 dB $\overline{n}$ = 6 - 100 ELEMENTS

Crossover Level (dB)	$\Delta\phi$	β	q^2	q^2 (dB)
0	0	1.0000	0.5000	3.0103
1	1.0441	0.7526	0.5706	2.4369
2	1.4582	0.5701	0.6369	1.9594
3	1.7643	0.4312	0.6987	1.5570
4	2.0164	0.3217	0.7566	1.2112
5	2.2326	0.2459	0.8026	0.9549
6	2.4307	0.1853	0.8437	0.7383
7	2.5929	0.1381	0.8786	0.5619
8	2.7370	0.1022	0.9073	0.4227
9	2.8812	0.0786	0.9280	0.3245
10	3.0074	0.0574	0.9458	0.2422

B. 4 BEAM DATA

TABLE 14. 4 BEAM UNIFORM - 20 ELEMENTS

Crossover (dB)	β_{1-2} [1]	$\Delta\phi_{1-2}$ [2]	β_{1-3} [3]	$\Delta\phi_{1-3}$ [4]	β_{1-4}	$\Delta\phi_{1-4}$	q^2 (dB)	q^2
6	1.00	0	1.00	0	1.00	0	6.02	0.25
1	0.61	9.48	-0.05	19.22	-0.20	29.59	2.76	0.53
2	0.33	13.29	-0.22	27.38	0.09	43.62	1.37	0.73
3	0.13	16.16	0.1[illegible]	33.83	0.11	56.62	0.51	0.89
7	0.61	18.50	-0.21	39.39	-0.01	72.16	9.48	0.97
6	-0.16	20.52	0.10	44.52	-0.09	91.85	1.13	0.77
6	-0.16	22.30	0.13	49.38	-0.09	94.81	1.49	0.71
7	-0.16	23.83	0.12	53.90	-0.04	97.19	[illegible].61	0.69
6	-0.21	25.25	0.09	58.57	0.02	99.29	1.55	0.70
9	-0.22	26.53	0.05	63.31	0.06	101.08	1.43	0.72
10	-0.22	27.67	0.02	69.23	0.08	102.60	1.31	0.74

Notes:

1. $\beta_{1-2} = \beta_{2-3} = \beta_{3-4}$

2. $\Delta\phi_{1-2} = \Delta\phi_{2-3} = \Delta\phi_{3-4}$

3. $\beta_{1-3} = \beta_{2-4}$

4. $\Delta\phi_{1-3} = \Delta\phi_{2-4}$

TABLE 15. 4 BEAM UNIFORM - 100 ELEMENTS

Crossover (dB)	$\beta_{1\text{-}2}$[1]	$\Delta\phi_{1\text{-}2}$[2]	$\beta_{1\text{-}3}$[3]	$\Delta\phi_{1\text{-}3}$[4]	$\beta_{1\text{-}4}$	$\Delta\phi_{1\text{-}4}$	q2(dB)	q^2
0	1.000	0	1.000	0	1.000	0	6.02	0.250
1	0.598	1.908	-0.056	3.819	0.[illegible]92	5.734	2.72	0.534
2	0.314	2.665	-0.215	5.336	0.092	8.018	1.31	0.740
[illegible]	0.121	3.206	-0.114	6.421	0.103	-9.656	0.51	0.890
[illegible]	0.018	3.638	0.010	7.292	0.018	10.975	0.13	0.971
[illegible]	-0.195	4.035	0.097	-8.081	-0.086	12.188	[illegible].13	0.771
6	-0.161	4.360	0.128	8.746	-0.081	13.184	1.50	0.708
[illegible]	-0.195	4.649	0.120	9.329	-0.033	14.073	1.58	0.695
8	-0.212	4.902	0.090	9.840	0.[illegible]20	14.854	1.52	0.70[illegible]
9	-0.217	5.119	0.054	10.279	0.055	15.525	1.41	0.722
10	-0.215	5.336	0.014	10.718	0.071	16.199	1.27	0.746

Notes:

1. $\beta_{1\text{-}2} = \beta_{2\text{-}3} = \beta_{3\text{-}4}$

2. $\Delta\phi_{1\text{-}2} = \Delta\phi_{2\text{-}3} = \Delta\phi_{3\text{-}4}$

3. $\beta_{1\text{-}3} = \beta_{2\text{-}4}$

4. $\Delta\phi_{1\text{-}2} = \Delta\phi_{2\text{-}3}$

TABLE 16. 4 BEAM COSINE (10 dB PEDESTAL) - 20 ELEMENTS

Crossover (dB)	β_{1-2} [1]	$\Delta\phi_{1-2}$ [2]	β_{1-3} [3]	$\Delta\phi_{1-3}$ [4]	β_{1-4}	$\Delta\phi_{1-4}$	q^2(dB)	q^2
0	1.00	0	1.00	0	1.00	0	6.00	0.25
1	0.49	11.20	0.20	22.85	-0.02	35.62	3.67	0.43
2	0.49	15.79	-0.01	32.96	0.01	54.70	2.52	0.56
3	0.33	19.30	-0.02	41.38	0.01	82.52	1.80	0.66
4	0.22	22.19	-0.00	49.05	-0.01	94.62	1.31	0.74
5	0.14	24.74	0.01	56.82	-0.01	98.54	0.92	0.81
6	0.0[illegible]	27.02	0.01	65.31	0.00	101.74	0.56	0.8[illegible]
4	0.01	29.06	0.01	76.26	0.01	104.39	0.32	0.93
6	0.01	30.93	0.00	91.01	-0.01	106.66	0.09	0.93
9	-0.00	32.66	-0.01	92.82	-0.00	108.64	0.04	0.99
10	-0.02	34.26	-0.01	94.39	-0.01	110.37	0.09	0.98

Notes:

1. $\beta_{1-2} = \beta_{2-3} = \beta_{3-4}$

2. $\Delta\phi_{1-2} = \Delta\phi_{2-3} = \Delta\phi_{3-4}$

3. $\beta_{1-3} = \beta_{2-4}$

4. $\Delta\phi_{1-3} = \Delta\phi_{2-4}$

TABLE 17. 4 BEAM COSINE (10 dB PEDESTAL) - 100 ELEMENTS

Crossover (dB)	β_{1-2}[1]	$\Delta\phi_{1-2}$[2]	β_{1-3}[3]	$\Delta\phi_{1-3}$[4]	β_{1-4}	$\Delta\phi_{1-4}$	q^2(dB)	q^2
0	1.000	0	1.000	0	1.000	0	6.02	0.250
1	0.697	2.197	0.178	4.396	-0.030	6.603	3.58	0.438
[illegible]	0.471	3.098	-0.025	6.204	-0.01[illegible]	9.329	2.42	0.572
3	0.313	3.747	-0.025	7.510	0.007	11.305	1.72	0.673
[illegible]	0.198	4.288	0.002	8.600	-0.013	12.962	1.20	0.758
[illegible]	0.123	4.721	0.016	9.475	-0.010	14.296	0.83	0.826
6	0.066	5.119	0.01[illegible]	10.279	0.003	15.525	0.51	0.889
[illegible]	0.027	5.480	0.011	11.012	-0.01[illegible]	16.649	0.24	0.946
8	0.001	5.4[illegible]	0.001	11.673	0.009	17.667	0.04	0.991
9	-0.015	6.096	-0.007	12.261	0.002	18.576	0.08	0.982
10	-0.024	6.349	-0.012	12.778	-0.004	19.375	0.13	0.970

Notes:

1. $\beta_{1-2} = \beta_{2-3} = \beta_{3-4}$

2. $\Delta\phi_{1-2} = \Delta\phi_{2-3} = \Delta\phi_{3-4}$

3. $\beta_{1-3} = \beta_{2-4}$

4. $\Delta\phi_{1-3} = \Delta\phi_{2-4}$

TABLE 18. 4 BEAM COSINE (15 dB PEDESTAL) - 20 ELEMENTS

Crossover (dB)	$\beta_{1\text{-}2}$[1]	$\Delta\phi_{1\text{-}2}$[2]	$\beta_{1\text{-}3}$[3]	$\Delta\phi_{1\text{-}3}$[4]	$\beta_{1\text{-}4}$	$\Delta\phi_{1\text{-}4}$	q^2 (dB)	q^2
0	1.00	0	[illegible].00	0	1.00	0	6.02	0.25
1	0.72	11.93	0.23	24.42	-0.01	38.33	3.77	0.42
2	0.51	16.87	0.01	35.49	0.00	60.56	2.68	0.54
3	0.36	20.64	-0.02	44.83	0.01	92.05	1.94	0.64
4	0.25	23.79	0.01	53.78	0.00	97.13	1.94	0.72
5	0.16	26.57	0.00	63.47	0.00	101.14	1.02	0.[illegible]5
6	0.11	29.06	0.01	76.26	0.00	104.39	0.71	0.85
7	0.06	31.31	0.01	91.41	0.01	107.10	0.46	0.90
8	0.03	33.44	0.00	93.59	0.00	109.49	0.22	0.95
9	0.01	35.36	0.00	95.43	-0.00	111.51	0.09	0.98
10	0.00	37.19	-0.00	97.09	-0.00	113.32	0.00	1.00

Notes:

1. $\beta_{1\text{-}2} = \beta_{2\text{-}3} = \beta_{3\text{-}4}$

2. $\Delta\phi_{1\text{-}2} = \Delta\phi_{2\text{-}3} = \Delta\phi_{3\text{-}4}$

3. $\beta_{1\text{-}3} = \beta_{2\text{-}4}$

4. $\Delta\phi_{1\text{-}3} = \Delta\phi_{2\text{-}4}$

TABLE 19. 4 BEAM COSINE (15 dB PEDESTAL) - 100 ELEMENTS

Crossover (dB)	β_{1-2}[1]	$\Delta\phi_{1-2}$[2]	β_{1-3}[3]	$\Delta\phi_{1-3}$[4]	β_{1-4}	$\Delta\phi_{1-4}$	q^2(dB)	q^2
0	1.000	0	1.000	0	1.000	0	6.02	0.250
[illegible]	0.709	2.341	0.205	4.685	-0.017	7.038	3.68	0.429
2	0.505	3.242	0.003	6.494	0.002	9.767	2.60	0.549
3	0.345	3.963	-0.024	7.945	0.005	11.967	1.87	0.650
4	0.237	4.505	-0.004	9.037	-0.004	13.628	1.38	0.727
6	0.153	5.010	0.005	10.060	0.005	15.189	0.97	0.800
6	0.093	5.444	-0.004	10.938	-0.004	16.537	0.64	0.862
4	0.093	5.806	0.002	11.673	-0.004	17.667	0.40	0.913
8	0.024	6.168	0.003	12.409	0.002	18.804	0.18	0.960
9	0.003	6.494	-0.002	13.073	-0.001	19.834	0.02	0.996
10	-0.008	6.748	-0.004	13.591	-0.002	20.640	0.04	0.990

Notes:

1. $\beta_{1-2} = \beta_{2-3} = \beta_{3-4}$

2. $\Delta\phi_{1-2} = \Delta\phi_{2-3} = \Delta\phi_{3-4}$

3. $\beta_{1-3} = \beta_{2-4}$

2. $\Delta\phi_{1-3} = \Delta\phi_{2-4}$

TABLE 20. 4 BEAM TAYLOR 20 dB $\bar{n}$ = 4 - 20 ELEMENTS

Crossover (dB)	$\beta_{1\text{-}2}$[1]	$\Delta\phi_{1\text{-}2}$[2]	$\beta_{1\text{-}3}$[3]	$\Delta\phi_{1\text{-}3}$[4]	$\beta_{1\text{-}4}$	$\Delta\phi_{1\text{-}4}$	q^2(dB)	q^2
0	1.00	0.04	1.00	0.07	1.00	0.11	6.02	0.25
1	0.68	17.93	0.15	21.22	-0.01	32.88	3.47	0.45
2	0.10	14.70	-0.01	30.51	0.04	49.60	2.37	0.96
3	0.29	17.93	0.01	38.01	-0.01	67.46	1.67	0.68
4	0.17	20.60	0.04	44.73	-0.06	98.89	1.13	0.77
5	0.10	22.89	0.04	51.06	-0.04	95.74	.76	0.84
6	0.68	28.48	0.02	57.61	0.00	98.89	.18	0.84
7	0.68	26.82	0.00	64.46	-0.04	101.47	.18	0.96
8	1.00	28.48	0.04	72.51	0.04	103.66	.22	0.96
9	0.68	30.05	-0.05	90.06	0.02	105.61	.32	0.93
10	-0.01	31.48	-0.06	91.59	0.00	107.30	.27	0.94

Notes:

1. $\beta_{1\text{-}2} = \beta_{2\text{-}3} = \beta_{3\text{-}4}$

2. $\Delta\phi_{1\text{-}2} = \Delta\phi_{2\text{-}3} = \Delta\phi_{3\text{-}4}$

3. $\beta_{1\text{-}3} = \beta_{2\text{-}4}$

4. $\Delta\phi_{1\text{-}3} = \Delta\phi_{2\text{-}4}$

TABLE 21. 4 BEAM TAYLOR 20 dB $\bar{n}$ = 4 - 100 ELEMENTS

Crossover (dB)	β_{1-2}[1]	$\Delta\phi_{1-2}$[2]	β_{1-3}[3]	$\Delta\phi_{1-3}$[4]	β_{1-4}	$\Delta\phi_{1-4}$	q^2 (dB)	q^2
0	1.000	0.038	1.000	0.072	1.000	0.108	6.02	0.25
[illegible]	0.676	2.089	0.157	4.180	0.008	6.27	3.47	0.45
2	0.449	2.917	0.007	[illegible].032	-0.063	8.782	2.44	0.57
[illegible]	0.292	3.530	0.029	7.074	-0.011	10.645	1.74	0.67
4	0.184	4.035	0.058	0.058	-0.063	4.180	1.25	0.75
5	0.110	4.469	0.058	8.965	-0.044	13.517	0.86	0.82
6	0.061	4.866	0.032	9.767	0.005	14.742	0.51	0.[illegible]5
4	0.033	5.191	0.000	10.425	0.034	15.750	0.27	0.94
8	0.017	5.480	-0.028	11.012	0.038	16.650	0.18	0.96
9	0.008	5.770	-0.051	11.600	0.022	17.553	0.22	0.95
10	0.006	6.023	-0.062	12.114	-0.002	18.348	0.27	0.94

Notes:

1. $\beta_{1-2} = \beta_{2-3} = \beta_{3-4}$
2. $\Delta\phi_{1-2} = \Delta\phi_{2-3} = \Delta\phi_{3-4}$
3. $\beta_{1-3} = \beta_{2-4}$
4. $\Delta\phi_{1-3} = \Delta\phi_{2-4}$

TABLE 22. 4 BEAM TAYLOR 30 dB $\bar{n}$ = 5 - 20 ELEMENTS

Crossover (dB)	$\beta_{1\text{-}2}$[1]	$\Delta\phi_{1\text{-}2}$[2]	$\beta_{1\text{-}3}$[3]	$\Delta\phi_{1\text{-}3}$[4]	$\beta_{1\text{-}4}$	$\Delta\phi_{1\text{-}4}$	q^2 (dB)	q^2
6	1.00	0	1.00	0	1.00	0	6.02	0.25
1	0.73	11.71	0.25	23.95	0.02	37.51	3.87	0.41
2	0.53	16.54	0.04	34.70	-0.01	58.64	2.76	0.[illegible]4
3	0.38	20.22	0.00	43.72	-0.01	91.32	2.08	0.62
1	0.27	23.32	-0.01	52.34	-0.01	96.41	1.55	0.77
6	0.19	26.05	-0.01	61.45	0.00	100.42	1.14	0.77
6	0.13	28.48	-0.01	72.51	-0.01	103.66	0.81	0.84
7	0.09	30.72	0.38	90.78	0.00	106.41	0.56	0.88
6	0.06	32.79	-0.01	92.95	-0.01	108.79	0.36	0.92
9	0.04	34.70	-0.01	94.81	-0.01	110.83	0.22	0.95
10	0.02	36.51	-0.01	96.49	-0.01	112.66	0.13	0.97

Notes:

1. $\beta_{1\text{-}2} = \beta_{2\text{-}3} = \beta_{3\text{-}4}$

2. $\Delta\phi_{1\text{-}2} = \Delta\phi_{2\text{-}3} = \Delta\phi_{3\text{-}4}$

3. $\beta_{1\text{-}3} = \beta_{2\text{-}4}$

4. $\Delta\phi_{1\text{-}3} = \Delta\phi_{2\text{-}4}$

TABLE 23. 4 BEAM TAYLOR 30 dB $\bar{n}$ = 5 - 100 ELEMENTS

Crossover (dB)	β_{1-2}[1]	$\Delta\phi_{1-2}$[2]	β_{1-3}[3]	$\Delta\phi_{1-3}$[4]	β_{1-4}	$\Delta\phi_{1-4}$	q2(dB)	q^2
0	1.000	0	1.000	0	1.000	0	6.02	0.250
[illegible]	0.733	2.377	0.270	4.758	0.042	7.147	3.89	0.408
2	0.542	3.314	0.067	6.639	-0.001	9.987	2.87	0.516
3	0.397	4.035	0.016	8.091	-0.010	12.188	2.18	0.605
4	0.294	4.613	0.004	9.256	-0.003	13.962	1.69	0.677
5	0.211	5.155	-0.003	10.352	0.007	15.637	1.27	0.746
6	0.157	5.589	-0.008	11.232	0.007	16.988	0.97	0.800
7	0.116	5.987	-0.010	12.040	0.000	18.234	0.72	0.848
8	0.087	6.349	0.004	12.778	-0.006	19.375	0.54	0.884
9	0.065	6.675	-0.006	13.443	-0.007	20.409	0.40	0.911
10	0.048	7.001	-0.002	14.110	-0.004	21.450	0.32	0.930

Notes:

1. $\beta_{1-2} = \beta_{2-3} = \beta_{3-4}$

2. $\Delta\phi_{1-2} = \Delta\phi_{2-3} = \Delta\phi_{3-4}$

3. $\beta_{1-3} = \beta_{2-4}$

4. $\Delta\phi_{1-3} = \Delta\phi_{2-4}$

TABLE 24. 4 BEAM TAYLOR 40 dB $\bar{n}$ = 6 - 20 ELEMENTS

Crossover (dB)	$\beta_{1\text{-}2}$[1]	$\Delta\phi_{1\text{-}2}$[2]	$\beta_{1\text{-}3}$[3]	$\Delta\phi_{1\text{-}3}$[4]	$\beta_{1\text{-}4}$	$\Delta\phi_{1\text{-}4}$	q^2 (dB)	q^2
0	1.00	0	1.00	0	1.00	0	6.97	0.25
1	0.75	12.78	0.29	26.25	0.03	41.57	3.98	0.40
2	0.56	18.08	0.16	38.37	-0.01	68.62	2.92	0.51
3	0.41	22.23	-0.00	49.16	-0.01	94.69	2.22	0.60
4	0.41	25.69	-0.01	60.12	-0.01	99.91	0.97	0.85
5	0.22	28.77	-0.01	74.28	-0.00	104.02	1.31	0.74
6	0.16	31.56	-0.01	94.69	-0.00	107.39	0.97	0.85
7	-0.01	34.17	-0.01	94.31	-0.00	110.28	0.71	0.85
8	-0.01	36.60	-0.01	96.57	-0.00	112.75	0.51	0.85
9	0.05	38.88	-0.01	98.54	-0.00	114.90	0.36	0.92
10	0.04	41.04	-0.01	100.28	-0.00	116.80	0.22	0.95

Notes:

1. $\beta_{1\text{-}2} = \beta_{2\text{-}3} = \beta_{3\text{-}4}$

2. $\Delta\phi_{1\text{-}2} = \Delta\phi_{2\text{-}3} = \Delta\phi_{3\text{-}4}$

3. $\beta_{1\text{-}3} = \beta_{2\text{-}4}$

4. $\Delta\phi_{1\text{-}3} = \Delta\phi_{2\text{-}4}$

TABLE 25. 4 BEAM TAYLOR 40 dB $\bar{n}$ = 6 - 100 ELEMENTS

Crossover (dB)	$\beta_{1\text{-}2}$[1]	$\Delta\phi_{1\text{-}2}$[2]	$\beta_{1\text{-}3}$[3]	$\Delta\phi_{1\text{-}3}$[4]	$\beta_{1\text{-}4}$	$\Delta\phi_{1\text{-}4}$	q2(dB)	q^2
6	1.000	0	1.000	0	1.000	0	6.02	0.250
1	0.753	2.629	0.306	5.263	0.054	7.909	3.01	0.397
2	0.570	3.675	6.675	7.364	-0.003	11.085	3.01	0.500
3	0.431	4.469	0.018	8.965	-0.002	13.517	2.36	0.584
4	0.322	5.155	-0.000	10.352	-0.000	15.637	1.82	0.658
5	0.246	5.697	-0.003	11.452	-0.001	17.327	1.44	0.717
6	0.185	6.204	-0.003	12.483	-0.002	18.918	1.13	0.771
2	0.138	6.675	-0.002	14.333	-0.002	20.409	0.87	0.819
6	0.102	7.110	-0.001	14.333	-0.001	21.798	0.66	0.859
9	0.078	7.473	-0.001	15.077	0.000	22.966	0.51	0.889
10	0.057	7.836	-0.000	15.824	0.001	24.144	0.39	0.915

Notes:

1. $\beta_{1\text{-}2} = \beta_{2\text{-}3} = \beta_{3\text{-}4}$

2. $\Delta\phi_{1\text{-}2} = \Delta\phi_{2\text{-}3} = \Delta\phi_{3\text{-}4}$

3. $\beta_{1\text{-}3} = \beta_{2\text{-}4}$

4. $\Delta\phi_{1\text{-}2} = \Delta\phi_{2\text{-}3}$

APPENDIX B. 4 BEAM RADIATION EFFICIENCY

This Appendix describes the radiation efficiency calculation for the case of 4 identical beams. For 4 beams, the determinant for the radiation efficiency factor becomes

$$\begin{vmatrix} (1-\lambda) & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & (1-\lambda) & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & (1-\lambda) & \beta_{34} \\ \beta_{41} & \beta_{42} & \beta_{43} & (1-\lambda) \end{vmatrix} = 0 \quad . \tag{49}$$

Recalling the symmetry of the coupling coefficients, this determinant can be written as

$$\begin{vmatrix} (1-\lambda) & \beta_{21}^{*} & \beta_{31}^{*} & \beta_{41}^{*} \\ \beta_{21} & (1-\lambda) & \beta_{32}^{*} & \beta_{42}^{*} \\ \beta_{31} & \beta_{32} & (1-\lambda) & \beta_{43}^{*} \\ \beta_{41} & \beta_{42} & \beta_{43} & (1-\lambda) \end{vmatrix} = 0 \quad . \tag{50}$$

The evaluation of the determinant analytically proceeds by cofactor expansion. Expanding and collecting terms leads to the final result of

$$\lambda^4 - 4\lambda^3 + (6-K)\lambda^2 + 2(K-A-2)\lambda + (2A+D+1) = 0 \quad . \tag{51}$$

The following definitions are made to simplify the expression:

$$\begin{array}{ll} \alpha = \beta_{21}\beta_{42}\beta_{41}^{*} & \gamma = \beta_{32}\beta_{43}\beta_{42}^{*} \\ \delta = \beta_{21}\beta_{32}\beta_{31}^{*} & \epsilon = \beta_{43}\beta_{31}\beta_{41}^{*} \\ \eta = \beta_{31}\beta_{42}\beta_{32}^{*}\beta_{41}^{*} & \sigma = \beta_{21}\beta_{42}\beta_{31}^{*}\beta_{43}^{*} \\ \rho = \beta_{21}\beta_{32}\beta_{43}\beta_{41}^{*} & \end{array} \tag{52}$$

$$\begin{aligned} K &= |\beta_{13}|^2 + |\beta_{32}|^2 + |\beta_{42}|^2 + |\beta_{21}|^2 + |\beta_{31}|^2 + |\beta_{41}|^2 \\ C &= |\beta_{21}|^2|\beta_{43}|^2 + |\beta_{42}|^2|\beta_{31}|^2 + |\beta_{32}|^2|\beta_{41}|^2 \end{aligned} \tag{53}$$

$$\begin{aligned} A &= Re[\alpha] + Re[\gamma] + Re[\delta] + Re[\varepsilon] \\ B &= Re[\eta] + Re[\rho] + Re[\sigma] \\ D &= C - 2B \end{aligned} \tag{54}$$

Since the coefficients of the fourth order polynomial obtained in (33) are real, the Fundamental Theorem of Algebra guarantees exactly four complex roots. Additionally, the roots of the polynomial correspond to the eigenvalues of the beam coupling matrix. Since the matrix is Hermitian, all eigenvalues (and therefore the polynomial roots) are guaranteed to be real. The largest root (dominant eigenvalue) is substituted into equation (38) to obtain the maximum radiation efficiency for the system.

APPENDIX C. COMPUTER SOURCE CODE

A. INTRODUCTION

The MATLAB™ (Matrix Laboratory) software package by The MathWorks© offers efficient implementation of the equations describing the coupling interactions and radiation efficiency. Since it is a programming language based on vector/matrix manipulation, it is fairly straightforward to enter the desired array coefficients, and have the computer calculate the solution. Additionally, many built in routines provide flexibility without the necessity of laboring to produce FORTRAN or PASCAL code. These features allow the programmer to concentrate primarily on the behavior and performance of the algorithm. The matrix nature of MATLAB also allows more efficient computation when data are presented in vector format. Acceptable throughput rates were obtained on a 386SX personal computer with an installed math co-processor.

For this thesis, the computer coding proceeded in a modular fashion. Principal tasks were coded as functions, with the main procedure calling the functions through arguments. Functions written included

1. Cosine on a Pedestal amplitude weight generator.
2. Taylor amplitude weight generator.
3. Beam coupling loss generator.
4. Array Pattern function

The source code for these functions is included in this Appendix. These functions were the principal calculational tools utilized in computing the raw data. Sufficient documentation is included within the function M-Files to allow straightforward analysis of program flow.

B. MATLAB SOURCE CODE

1. COSINE ON A PEDESTAL AMPLITUDE WEIGHTS

```
function y=cospedwn(N,p)
% function Y=COSPEDWN(N,p)
%
%
% This function calculates amplitude weights for a cosine on a pedestal
% window. The input arguments is the number of array elements, (N) and
% the pedestal size in dB. The function returns the amplitude coefficients.
%
%
clg;
ped=10.^(-p/20)
l=1:N;
xl=2*(l-1)/(N-1) -1;
y=(((1-ped)*abs(cos(xl*pi/2))) + ped).';
```

2. TAYLOR AMPLITUDE WEIGHTS

```
function a=taylor(slr,n_bar,N)
%
% function a=TAYLOR(slr,n_bar,N)
%
% This function calculates amplitude weight coefficients for an
% n element array based on a Taylor amplitude distribution. The
% input arguments are the desired main lobe - side lobe ratio (dB),
% the sidelobe parameter n_bar, and the number of array elements.
%
% The output variable is the amplitude weights an.
% The procedure is outlined in the text " The Handbook of Antenna
% Design (vol 2)" edited by A.W. Rudge, K. Milne, A.D. Olver, and
% P. Knight.
%
clg;
p=n_bar-1;
dbamp=20/log(10);
sll=exp(abs(slr)/dbamp);          % Calculate sidelobe level as a ratio
A=(1/pi)*log(sll+sqrt(sll.^2-1)); % Generate intermediate parameters
sigma=n_bar/(sqrt(A.^2 + (n_bar-1/2).^2));
f=ones(p,1);

% Begin calculation of amplitude weights
for m=1:p
```

```
  z(m)=sigma*sqrt(A.^2+(m-1/2).^2);
  K(m)=(fact(p)).^2 ./ (fact(p+m) .* fact(p-m));
end

for n=1:p
  P(n)=prod(1-(n.^2 ./ z.^2));
end
f=K.*P;
J=zeros(N/2,p);
for i=1:N/2
  for k=1:p
    J(i,k)=f(:,k).*cos(k.*pi.*2*(i-1)/N);
  end
end
g=1+2*sum(J.');
% Only 1/2 of the coefficients need to be calculated due to symmetry
a=[fliplr(g) g].';
a=a/max(a);             % Normalize the amplitude to max of 1
```

3. BEAM COUPLING LOSS CALCULATOR

```
function beta=couploss(N,crosslvl,win)
%
% function beta=COUPLOSS(N,crosslvl,win)
% This function calculates coupling losses for overlapping beams
% in a multiple beam antenna. The multiple beams are generated using
% a linear array with various choices for amplitude windows.
% A switch parameter is used to choose the amplitude weighting as follows
%       win == 1    Rectangular window
%       win == 2    Cosine on a pedestal (Pedestal size is input)
%       win == 3    Taylor    (Sidelobe level and n_bar are input)
%
clg;
% Initialize parameters
dB=crosslvl;
k=2*pi;
d=.5;
kd=k*d;
zeta=(N+1)/2;

% Get amplitude coefficients
if win == 1
  a1=boxcar(N);
  a2=a1.';
elseif win == 2
  p=.3199;
  a1=cospedwn(N,p);
  a2=a1.';
elseif win == 3
  slr=20;
  n_bar=4;
  a1=taylor(slr,n_bar,N);
  a2=a1.';
end
```

```
% Get crossover squint angles
level=10 .^(-dB/20);
u=0;
xi=1;
delta_u=1/10000;
a=a1(N/2+1:N).';
m=1:N/2;
Max=a*ones(10,1)+a*ones(10,1);

while xi >= level
   u=u+delta_u;
   z1=a*exp(j*(2*m-1)/2*kd*u).';
   z2=a*exp(-j*(2*m-1)/2*kd*u).';
   xi=1/Max*(z1+z2);
end
u_squint=u;
crossover_des=level;
crossover_comp=xi;

% Initialize scan angles and sums
up=kd*u_squint;
uq=-kd*u_squint;
du=uq-up;
dtheta=(acos(du/kd))*180/pi;

% Set up matrices and calculate summations
A1=a1*a2;
I=zeros(N,N);
Z1=0;Z2=0;Z3=0;
for m=1:N
   for n=1:N
      if n==m
         I(m,n)=2*kd;
         Z1=Z1+A1(m,n).*I(m,n).*exp(-j*m*du);
         Z2=Z2+A1(m,n).*I(m,n).*exp(j*0);
         Z3=Z3+A1(m,n).*I(m,n)*exp(j*0);
      else
         I(m,n)=2*((sin((n-m)*kd))/((n-m)));
         Z1=Z1+A1(m,n).*I(m,n).*exp(j*(up*m-uq*n));
         Z2=Z2+A1(m,n).*I(m,n).*exp(j*up*(m-n));
         Z3=Z3+A1(m,n).*I(m,n).*exp(j*uq*(m-n));
      end
   end
end
% Calculate coupling coefficient here
ex=1.*exp(j*zeta*du);
X=ex.*Z1;
Y=((Z2).^(.5)) .* ((Z3).^(.5));
b=X./Y;
beta=abs(b);
```

4. ARRAY PATTERN FUNCTION

```
function x=arrpat(a,psi_prime1,n)
%
% X=ARRPAT(a,psi_prime1,n)
%
% This function calculates the far-field array pattern for a
```

```
% linear array. Input arguments are the amplitude weight coefficients
% the desired steering angle off broadside, and the number of elements.
% The interelement spacing is automatically assumed to be lambda/2.
%
clg;
if psi_prime1 <= 90                    % Refer steering angle to broadside.
   psi1=(pi/2) - psi_prime1*(pi/180);
else
   psi1=psi_prime1*(pi/180);
end
[j,k]=size(a);
if j > k
   a=a.';
end
lambda=1;                  % Normalize wavelength
k=2*pi/lambda;
d=lambda/2;                % Interelement spacing
kd=k*d;
phi1= -kd*cos(psi1);       % Generate phase shift terms to steer the
                           % beam.
if rem(n,2) ~= 0
   z=-(n-1)/2:(n-1)/2;
else
   z=linspace(-n/2,n/2,n);
end
psi=linspace(0,2*pi,256);
e1=exp(j*z*phi1);                % Phase weighting terms
se=size(e1);
            % Generate amplitude and phase weighted terms
a1=a(n/2+1:n);
X1=zeros(256,n/2);
for l=1:256
   for z = 1:n/2
      X1(l,z)=(a1(:,z).*cos(kd.*cos(psi(l)).*(2*z-1)/2+((2*z-1)/2.*phi1)));
   end
end
m1=2*abs(sum(X1.'));             % Sum the elements to get the array factor
x=m1/max(m1);
```

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